

Numerical Investigation of the Turbulent Unsteady Flow and Noise Generation in an Iced Airfoil

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Abstract

The icing phenomenon modifies the external shape of the aircraft and negatively affects its aerodynamic performance and stability. In particular, glaze ice influences aerodynamic performance significantly due to the generation of complex flows that include separation bubbles. In this study, we predicted the unsteady aerodynamics and noise generation of an airfoil with a glaze ice shape using the lattice-Boltzmann method (LBM). The LBM simulation of the GLC305 airfoil with a dual-horn ice shape was performed at different angles of attack, and the predictions were compared against the experimental data for validation purposes. The noise analysis was performed using the permeable Ffowcs Williams–Hawking's acoustic analogy based on the aerodynamic analysis results. The flow structures of the ice-induced separation bubble were analyzed considering instantaneous turbulent features, wherein unsteady flow phenomena, such as Kelvin-Helmholtz instability, recirculation zones, and shear layer roll-up, were observed. Furthermore, the noise analysis results showed an increase in the noise levels at low-to-mid frequencies owing to the separation bubble and thick boundary layer generated by the ice shape, inducing maximum noise in the low-frequency range. Finally, we captured a dipole-like directivity with an increasing angle of attack; the noise propagated toward the ice shape for lower angles of attack.

Keywords: Iced airfoil, Turbulent flow, Aerodynamic Noise, Aeroacoustics, Computational Fluid Dynamics, Lattice-Boltzmann method

1. Introduction

Aircraft icing primarily occurs when an aircraft passes through regions of relatively high humidity and below-freezing temperatures. Aircraft icing affects the leading edges of low-curvature surfaces, such as wings, horizontal stabilizers, and windshields. Icing is broadly classified into three types: rime ice, glaze ice, and mixed ice. Rime ice forms when droplets strike the aircraft surface at low temperatures and freeze immediately, creating a rough surface with a streamlined shape. In contrast, glaze ice, which is also referred to as clear ice, forms near the freezing point, where droplets hitting the aircraft surface do not freeze immediately but form a thin water film, resulting in ice that forms slowly and irregularly into a horn shape. Glaze ice is particularly dangerous because it creates a more streamlined and heavier ice layer, significantly altering the shape of airfoil and aerodynamics. Mixed ice is a combination of rime and glaze ice, forming when varying droplet sizes freeze at different rates, leading to complex ice structures that can severely degrade the aerodynamic performance of the airfoil [1-3]. Ice accretion on a lifting surface alters the surface roughness and the external shape, negatively impacting aerodynamic performance, flight stability, and aircraft safety. Additionally, icing on the main and tail wings could significantly increase the drag, reduce the maximum lift coefficient, decrease the angle of stall onset, and degrade the effectiveness of the control surface [4, 5]. The accumulation of ice on rotor blades and engine intake systems can degrade rotorcraft performance in terms of reduced efficiency of rotor blades, changes in vibration characteristics, and engine flameout caused by the ingestion of ice particles. These cumulative effects often raise significant safety concerns [6–8]. Recent developments in electric-powered urban air mobility (UAM) vehicles designed for operation in urban environments have increased icing-related safety problems. Particularly, UAM vehicles that fly at low altitudes over highly populated urban areas are more likely to encounter atmospheric conditions within the temperature and humidity ranges that favor icing. Furthermore, low-altitude urban environments may include microclimates with high moisture content due to proximity to bodies of water or the heat island effect, making them more susceptible to icing. Moreover, flying over urban area may exposure to a variety of climates, which increases the likelihood of encountering icing conditions. The electrical energy required for eliminating ice accretion could constrain the operational capacity of UAM vehicles that use electric batteries rather than internal combustion engines. Therefore, the limited availability of electrical energy on these vehicles makes it challenging to operate anti- and de-icing devices, making them more susceptible to icing compared to traditional rotorcraft.

Ice accumulation has a significant impact on both the aerodynamic performance and flow-field characteristics of an airfoil. The resulting changes in flow separation, wake dynamics, and surface pressure distribution not only degrade aerodynamic efficiency but also influence the mechanisms of aerodynamic noise generation. However, most previous studies have primarily focused on analyzing the effect of ice accretion on airfoil geometry and aerodynamic performance, while limited attention has been given to its implications for aeroacoustic behavior. Poinsatte [9] and Kim and Bragg [10] offered fundamental experimental data on the aerodynamic degradation caused by glaze and horn-shaped ice formations. Krøgenes et al. [11], Zhang et al. [12], and Ferdous and Haider [13] conducted numerical and experimental analysis on various airfoil configurations, demonstrating that mixed-ice conditions and runback water effects significantly modify boundary-layer development and stall characteristics, leading to lift reduction and drag increase. More recently, high-fidelity computational approaches such as wall-modeled large eddy simulation (WMLES) and roughness-resolving methods have been applied to better capture the complex flow over iced surfaces [14, 15]. As mentioned earlier, the icing phenomenon adversely affects the aerodynamic performance of aircraft, posing significant safety concerns. Therefore, various ice protection systems have been proposed to either remove ice from aircraft or prevent ice formation, and they are indispensable components for ensuring safe operation. For instance, hot-air anti-icing systems utilize the bleed air from the engine to mitigate the occurrence of icing. The electrothermal anti-/de-icing method can overcome the limitations of hot-air anti-icing systems, offering energy-efficient solutions to prevent the formation of icing on aircraft effectively. However, detecting the occurrence of icing is essential for achieving optimal operation because removing ice from the aircraft consumes energy [16, 17]. The acoustic sensor, which involves discerning alterations in acoustic characteristics resulting from icing patterns to predict the possibility of ice formation, has gained substantial interest in recent years owing to the elimination of the need to attach sensors to aircraft surfaces; however, the variations in noise sources and acoustic features induced by icing should be identified to enhance the reliability of this approach [18, 19].

Detecting the occurrence of icing through acoustic methods requires a fundamental understanding of how icing alters the aerodynamic noise characteristics of an airfoil. The interaction between the turbulent boundary layer and the trailing edge is known to generate self-noise, where eddies formed near the sharp trailing edge induce surface pressure fluctuations that radiate as sound. When icing occurs, the presence of surface roughness, the development of turbulent separation bubbles, and enhanced turbulence intensity further modify this mechanism, leading to an increase in noise level. Several studies have examined the influence of icing on airfoil aerodynamics [20] and the dynamics of turbulent separation bubble (TSB) bursting [21, 22], but the relationship between these

flow phenomena and the resulting acoustic characteristics remains insufficiently understood. Several studies have investigated the aeroacoustic behavior of iced airfoils through both numerical simulations and experiments. Szasz et al. [23] performed computational fluid dynamics (CFD) and acoustic analyses using the inhomogeneous wave equation to compare the noise levels between clean and iced airfoils. Bao et al. [24] and Gil and Lee [25] investigated the effects of ice formation parameters on the flow field, pressure variations, and separation characteristics. They further compared the noise characteristics using the acoustic analogy based on the Ffowcs Williams-Hawkings (FW-H) equation and observed an increase in noise compared with a clean surface. Hwang et al. [26] experimentally validated this trend for the S809 airfoil, showing a 2–4 dB increases in overall sound pressure level (SPL) as the size of the leading-edge rime ice increased. Caicedo and Virk [27] utilized three different turbulence models, including Reynolds-averaged Navier-Stokes (RANS), detached eddy simulation (DES), and large eddy simulation (LES), to predict the aeroacoustic response of iced airfoils. The comparative analysis showed that boundary layer separation induced by surface ice is the main contributor to elevated noise levels. Xiao and Tong [28] performed noise measurements on a NACA0012 airfoil, considering two different ice accretion thicknesses at various angles of attack and flow velocities. They observed an increase in noise magnitude with the increasing ice thickness, angle of attack, and velocity; the measured results were compared with the LES simulation data.

From the above literature, previous studies have focused either on aerodynamic degradation or studying the impact of ice accretion on the noise levels. To date, the contribution of unsteady flow features, particularly TSB induced by glaze ice, to the resulting noise generation at various angles of attack has not been sufficiently explored. In this study, the lattice-Boltzmann method (LBM) was applied to simulate the development process of TSB induced by the horn-shaped ice formed on the leading edge of an airfoil at various angles of attack, including pre- and post-stall conditions. Moreover, an acoustic analogy based on the FW-H equation with permeable surfaces was employed to assess the ice-induced noise at far-field. The flow features of the ice-induced TSB, including the unstable shedding shear layer, flow separation and reattachment on the surface of the airfoil, and the turbulent flow field around the iced airfoil, were analyzed in detail. A spectral analysis was performed to examine the variations in the acoustic characteristics by varying the angle of attack, and the directivity of ice-induced noise was predicted to study the influence of icing on noise generation. The findings in this study can support the development of acoustic-based ice detection systems by identifying frequency and directional characteristics of ice-induced noise.

2. Numerical Methods

2.1 Lattice-Boltzmann Method

PowerFLOW, the LBM-based commercial solver, was employed to compute the turbulent flow field around the iced airfoil. The advantages of LBM include its inherently transient nature, efficient parallel computation, and robust handling of complex geometries, particularly for horn-shaped ice, rendering it a suitable simulation tool for computational aeroacoustics [29-33]. In comparison with conventional CFD methods used for solving continuum-based Navier–Stokes equations, the LBM numerically solves the discrete kinetic energy equation to simulate the collision and propagation of particles of a fluid on a discretized lattice and calculates the macroscopic properties of the flow using the particle distribution function [34, 35]. PowerFLOW integrates the Bhatnagar–Gross–Krook (BGK) collision model with a D3Q19 lattice structure to resolve both large- and small-scale flow features, providing a balance between accuracy and computational efficiency [36, 37].

The lattice-Boltzmann equation based on the time-extrinsic advection equation can be expressed as follows:

$$f_i(\mathbf{x} + \mathbf{v}_i \Delta t, t + \Delta t) - f_i(\mathbf{x}, t) = C_i(\mathbf{x}, t) \quad (1)$$

where f_i denotes the particle distribution function in the i -th direction; $\mathbf{v}_i$ represents the particle velocity; and $\mathbf{v}_i \Delta t$ and Δt indicates the spatial and temporal changes, respectively. The left-hand side of Eq. (1) represents the equations for the advection of particles, whereas the right-hand side term denotes the change in the f_i function caused by the collision between particles. The lattice-Boltzmann equation based on the time-extrinsic advection equation includes the collision term C_i , which can be expressed using the BGK collision model [38], as in Eq. (2).

$$C_i(\mathbf{x}, t) = \frac{\Delta t}{\tau} [f_i(\mathbf{x}, t) - f_i^{eq}(\mathbf{x}, t)] \quad (2)$$

where τ denotes a dimensionless coefficient of the time required for particles to stabilize after collision, and its relationship with the kinematic viscosity coefficient ν can be defined as

$$\nu = \left(\tau - \frac{1}{2}\right) T \quad (3)$$

In Eq. (2), f_{eq} denotes a local equilibrium distribution function, T refer to temperature which can expressed as indicated in Eq. (4) for a D3Q19 model:

$$f_i^{eq} = \omega_i \rho \left[1 + 3(\mathbf{v} \cdot \mathbf{u}) + \frac{9}{2}(\mathbf{v} \cdot \mathbf{u})^2 + \frac{3}{2}(\mathbf{u} \cdot \mathbf{u}) \right] \quad (4)$$

where $\mathbf{u}$ indicates the macroscopic velocity, $\mathbf{v}$ represents the discrete lattice velocity direction and ω_i represents the weight in the i -th direction with the values

$$\omega_i = \begin{cases} \frac{2}{36} & i = 1, 2, \dots, 6 \\ \frac{1}{36} & i = 7, 8, \dots, 18 \\ \frac{12}{36} & i = 19 \end{cases} \quad (5)$$

Turbulence modeling in PowerFLOW was carried out using a modified form of the renormalization group (RNG) k - ϵ model, which enhances the LBM framework by improving the prediction of boundary layer separation and turbulent mixing. The model adjusts local viscosity to better capture small-scale turbulence and has been successfully integrated into the LBM approach in previous studies [39–41]. PowerFLOW simulation is conceptually similar to hybrid RANS/LES approaches. The turbulence modeling approach employed in the PowerFLOW solver is called LBM-Very Large Eddy Simulation (LBM-VLES) as the attached boundary layer are modeled to consider the impact of turbulence, while turbulent fluctuations caused by flow separation are resolved [42, 43]. It includes the wall modeling techniques to capture the effects of boundary layers on aerodynamic surfaces for high Reynolds number flows. These techniques ensure that the simulations efficiently predict shear stress and pressure distribution along surfaces. Moreover, the wall modeling enables to simulate near-wall flows without requiring excessively fine mesh resolutions. PowerFLOW includes advanced wall modeling techniques to capture the effects of boundary layers on aerodynamic surfaces [44]. The wall boundary conditions are imposed on the surface of the body to model the interaction between the fluid particle and solid surface, such as the particle bounce-back process for a no-slip wall and the specular reflection for a slip wall, respectively. If the bounce back boundary condition is applied to the surfaces, the fluid particles hitting a wall boundary are reflected back, ensuring zero velocity at the wall [45]. It should be noted that, while the use of wall modeling in LBM facilitates efficient simulation of high Reynolds number flows, it may not fully resolve fine-scale turbulence structures near the wall. Moreover, the Cartesian-grid-based voxel mesh, although capable of representing complex geometries through surface definitions, may introduce discretization errors in regions with sharply curved or thin features. These limitations should be taken into account when interpreting near-wall turbulent behavior in the present results.

2.2 Ffowcs-Williams and Hawkings (FW-H) Acoustic Analogy

Although the inherent unsteady and compressible nature of the LBM solver makes it suitable for directly obtaining acoustic fluctuations, numerous grid cells are required to resolve the frequencies of interest. This poses significant computational challenges despite adopting parallel computation. As the direct computation of far-field noise is impractical owing to grid refinement restrictions, this study implemented a hybrid method by combining a high-fidelity flow simulation tool and an acoustic solver to predict the far-field noise radiated from an iced airfoil. This method facilitated the efficient sequential computation of the flow field (acoustic source) and acoustic field with different governing equations and numerical approaches by assuming that the influence of the acoustic field on the flow field can be ignored. In this study, the flow simulations over the iced airfoil were performed using the transient LBM-VLES approach, and the far-field noise was computed using Ffowcs-Williams and Hawkings (FW-H) acoustic analogy based on Farrasat's formulation 1A, extended for permeable surface integration to avoid high computational costs. The formulations are implemented in the time domain, using the source-time dominant algorithm.

The FW-H acoustic analogy was extended from Lighthill's analogy to include moving surface boundaries comprising monopole and dipole source distributions and the contribution of the quadrupole source. However, evaluating quadrupole sources contributions to noise radiation is still expensive computationally because a volume integral over the entire region enclosing the noise sources is needed. The FW-H method with permeable integration surface is useful way for predicting far-field noise generated by monopole, dipole, and quadrupole effectively by replacing the volume integral with surface integral over a permeable surface that encloses the noise sources. In this study, a permeable FW-H surface is defined encompassing the iced airfoil and the unsteady flow properties are stored on the permeable surface to perform surface integration for the far-field noise calculation. However, the boundaries of the FW-H integration surface, especially in the downstream, contact with the vortical structures from the wake of the iced airfoil that results in pseudo-sound due to hydrodynamic fluctuations [46]. The downstream faces of the permeable surface should be located far from the iced airfoil to eliminate the effects of spurious source of sound. Creating the large size of the permeable surface to avoid the intersection of FW-H surface with the separated flow could cause high computational costs. Efforts have been made to reduce spurious wave contamination when vortical structures pass across the FW-H integration surface. A corrective approach was proposed by Wang et al. [47] and Shur et al. [48] closely placed multiple surfaces to record data that is averaged for the final calculation. Spalart et al. [49] adopted the averaging strategy to predict the landing gear noise, and Mendez et al. [50] explored the number and positions of the FW-H surfaces for averaging. In this study, end-caps with five layers of planar surfaces are located at the boundaries of FW-H surface in the downstream

direction to filter pseudo-sound contamination, and they are placed close enough to ignore the temporal lag of acoustic signal. Phase-averaging the far-field noise signals allow filtering the pseudo-sound signal caused by the vortical structures passing through the integration surface, as suggested by Shur et al. [48, 51].

Owing to low dispersion and dissipation [52], the LBM accurately resolves near-field unsteady structure, which serves as reliable input for the FW-H acoustic analogy. This enables high-fidelity prediction of far-field noise through surface integration without significant contamination from numerical errors. Because the accuracy of FW-H predictions strongly depends on the fidelity of the aerodynamic source data, particular attention was given to verifying the unsteady pressure and velocity fields that serve as input for the acoustic computation. The aerodynamic and flow-field validations of the present PowerFLOW LBM solver, including the lift, drag, surface-pressure coefficients, and boundary-layer velocity profiles, are presented in Section 3. These results showed good agreement with experimental data, providing fundamental verification of the input flow field used in the aeroacoustic analysis. It should also be noted that for clean airfoils, aerodynamic noise generation is generally relatively low; however, noticeable broadband noise may still arise under specific flow conditions, such as transitional separation or trailing-edge vortex shedding. Consequently, reliable experimental acoustic data suitable for direct validation are extremely limited. For this reason, the verification of aerodynamic and flow-field accuracy is considered a practical and effective validation approach for assessing the reliability of the present noise analysis. Moreover, the hybrid approach combining flow-field data computed by PowerFLOW with the permeable FW-H method has been extensively validated in previous aeroacoustic investigations involving airfoil [53], turbofan [40], and cylinder [54]. These studies have demonstrated that the combination of the lattice-Boltzmann-based flow solver and the permeable FW-H formulation provides accurate broadband noise predictions.

2.3 Iced Airfoil Model

In this study, a GLC305 airfoil with a glaze ice shape, including a dual horn (denoted as the 944 ice shape), obtained from the NASA Icing Research Tunnel [55], was considered the computational configuration. The icing conditions for this specific ice shape, reported in FAR Part 25 Appendix C, include a liquid water content (LWC) of 0.54 g/m^3 , a median volumetric diameter (MVD) of $20 \text{ }\mu\text{m}$, an angle of attack of 4° , and an icing exposure time of 22.5 min. Table 1 lists the detailed conditions and Fig. 1 depicts the shape of the iced airfoil used for the aeroacoustic analysis. The probes depicted in Fig. 1 indicate their placements to measure the development of boundary layer flow over the iced airfoil and turbulence kinetic energy (TKE) at the span center position of $z/c = 0.5$. These measurements of the turbulent flow field around the suction side of the iced airfoil were utilized for

LBM solver validation. The specific locations, in the chordwise and normal directions were $x/c = 0.12, 0.3, 0.45, 0.55$ and $y/c=0.1$ where x/c and y/c represent the non-dimensional chordwise and normal positions.

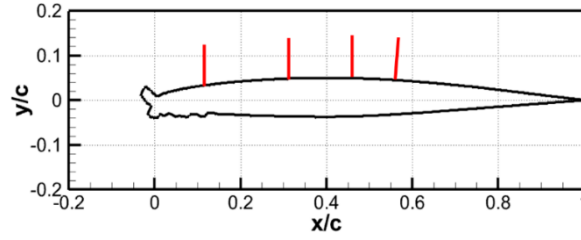


Fig. 1. Geometry of the iced airfoil and the probe locations.

Table 1. Icing and airfoil configuration parameters.

Parameter	Value
Airfoil	GLC305
Chord length, c [m]	0.9144
Span length, l [m]	0.2743
Angle of attack, α [°]	4
LWC [g/m ³]	0.54
MVD [μ m]	20
Exposure time [min.]	22.5

2.4 Simulation Setup

The overall computational domain and boundary conditions are depicted in Fig. 2. The computational domain for simulating iced airfoil extends from 5 times chord lengths upstream to 20 times chord lengths downstream, with a lateral extent of 30% of chord lengths in the spanwise direction [56]. This ensures that any flow disturbances or vortices generated by the iced airfoil are fully captured and dissipated before reaching the boundaries. The inlet and outlet conditions were applied to the surfaces of the far-field domain, and a nonslip condition was imposed on the airfoil surface. Moreover, periodic boundary conditions were implemented on both sides of the far-field domain along the spanwise direction to mitigate three-dimensional influences. The surface mesh and the placement of FW-H surface are depicted in Fig. 3. The permeable FW-H integration surface (denoted with the blue color) is positioned to enclose the iced airfoil and capture the relevant noise-generating flow features. The location was determined based on preliminary simulations to ensure it effectively captures the dominant acoustic sources while minimizing numerical error. End-caps of planar surfaces (denoted with yellow color) are placed at the boundaries of permeable surface in the downstream direction for averaging far-field acoustic signals that allow

eliminating pseudo-sound contamination. PowerFLOW spatially discretizes the computational domain into cubic elements called voxels. Local refinements can be defined using voxel resolution regions, where the refinement of two adjacent regions can vary by a factor of two. Surface facets, referred to as surfels, are defined at the intersection of cubic voxels and solid geometries. These definitions of voxels and surfels enable complex geometries, such as ice accretion on wing structures, to be accurately discretized within the domain. In this study, the surface grid was refined near the leading and trailing edges of the airfoil to accurately represent the dual-horn ice shape and capture the high velocity and pressure gradients. Furthermore, a Cartesian grid topology was used for the volume discretization to predict the formation of the TSB and the near-field of the vortical structures shed from the leading-edge ice accretion on the airfoil. The total number of grid cells used in this study was approximately 21 million, and the minimum grid size employed was 1.3 mm corresponding to a chord length of 0.14%. As mentioned previously, the spatial grid density in PowerFLOW increases by a factor of two based on the minimum grid size with the increase in the variable resolution (VR) level. In this study, the computational domain was divided into seven VR regions. The finest grids were deployed near the iced airfoil region, and sponge zones were applied near the boundaries of the grids to prevent the reflection of vortical structures and acoustic waves into the computational domain. As mentioned previously, the relevant particle interactions is introduced into the collision term at the solid–fluid boundary for computational efficiency and application of various wall conditions [34]. PowerFLOW primarily uses a bounce back process for enforcing no slip conditions at walls. This is common method in LBM where particles hitting a wall boundary are reflected back, effectively ensuring zero velocity at the wall [45].

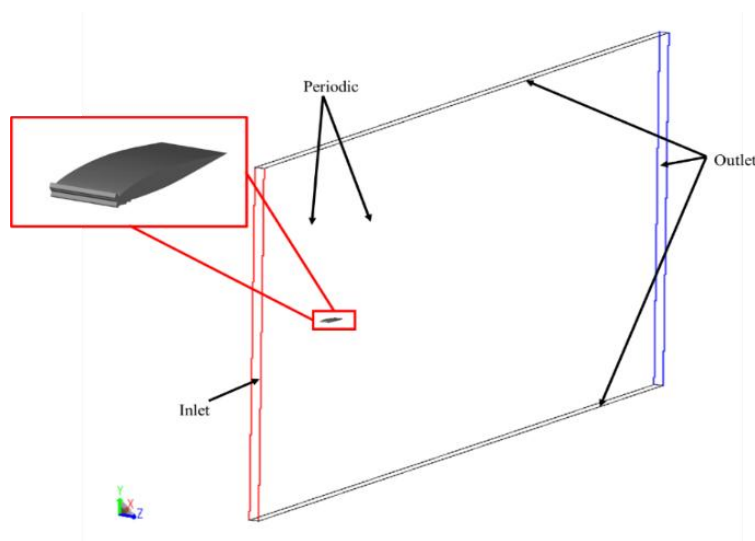
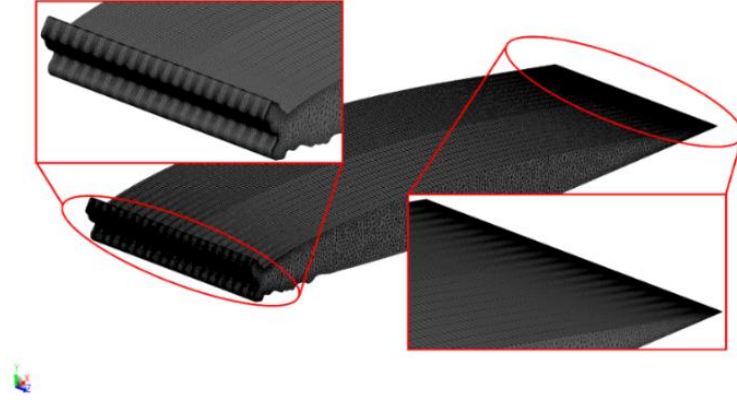
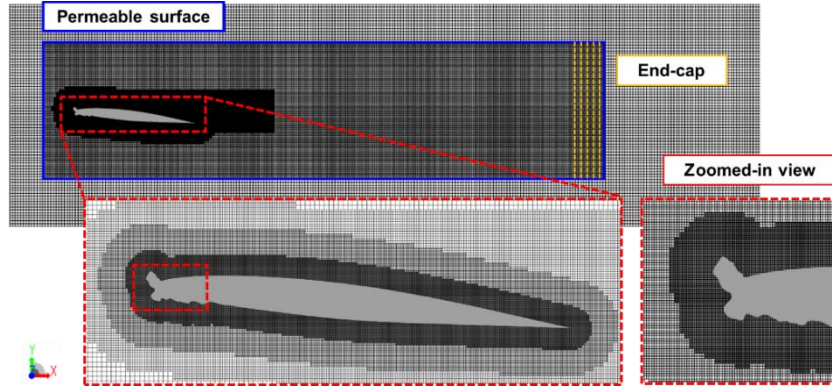


Fig. 2. Computational domain for LBM simulations



(a) Surface mesh on iced airfoil



(b) VR region with permeable surface with end-cap

Fig. 3. Mesh used for the iced airfoil simulation.

The aerodynamic analysis of the iced airfoil was conducted at a Mach number of 0.12 and Reynolds number of 3,500,000; Table 2 lists the detailed simulation conditions. The LBM simulation was conducted for 0.7 s with a time step of 2.28×10^{-6} s, corresponding to a non-dimensional time step of 1.02×10^{-4} . The time step was carefully chosen to ensure that the Courant-Friedrichs-Lewy (CFL) condition was satisfied, typically maintaining a CFL number below 1 across the computational domain to ensure numerical stability and accuracy. The computational analysis was conducted using parallel calculations executed on an Intel Xeon Gold 6230R CPU operating at 2.1 GHz with 50 cores, and the elapsed simulation was approximately 24 h. The aerodynamic data on permeable surfaces enclosing the noise sources of the iced airfoil were accumulated to evaluate the averaged results and far-field noise after 0.55 s. For the noise data processing, a Hanning window function was applied to the time-series data before obtaining the acoustic spectrum through the fast Fourier transform (FFT). This windowing technique helps to reduce spectral leakage, ensuring more accurate frequency domain representations. The window averaging was conducted over 7 periods to smooth the data and minimize the influence of transient effects. The

sample bin width was selected based on the Nyquist criterion, ensuring that the frequency resolution was sufficient to capture the relevant acoustic phenomena without introducing aliasing. The sample bin width is 5.945 Hz.

Table 2. LBM Simulation parameters for iced airfoil

Parameter	Value
Mach number, M [-]	0.12
Reynolds number, Re [-]	3,500,000
Finest voxel size [mm]	1.3
Total simulation time [s]	0.7
Time step [s]	2.28×10^{-6}

3. Results and Discussions

3.1 Aerodynamic Force and Velocity Field

Prior to conducting the aeroacoustic analysis of iced airfoil, the aerodynamic performance of the clean GLC305 airfoil was examined to validate the predictive capability of the present LBM solver. In Fig. 4, the computed lift and drag coefficients were compared with the experimental data over angles of attack ranging from 0° to 8° . The predicted lift coefficients ($C_l = 0.066, 0.286, 0.500$, and 0.910) showed close agreement with the measured data ($C_l = 0.084, 0.275, 0.480$, and 0.886), confirming that the LBM solver accurately captures the lift characteristics of the clean airfoil. However, the drag coefficients were slightly overpredicted ($C_d = 0.0095, 0.0110, 0.0147$, and 0.0317) compared with the experimental results ($C_d = 0.0049, 0.0055, 0.0082$ and 0.0123), which can be attributed to the limited near-wall resolution in the present LBM simulation. With the solver validated for the clean airfoil, the LBM predictions of the lift, drag, and surface pressure coefficients of the iced airfoil, as well as the velocity profiles inside the turbulent flow field induced by leading-edge ice accretion, were compared with the experimental results. The GLC305 iced airfoil with a dual-horn shape was evaluated in the NASA Langley Low Turbulence Pressure Tunnel (LTPT) [57]. Figs. 5(a) and 5(b) present a comparative analysis of the lift and drag coefficients of the iced airfoil at different angles of attack, respectively, against the measurements and simulation results obtained from zonal detached eddy simulation (ZDES) and RANS [58]. The validation results showed that the variation in the lift coefficients with the angle of attack obtained from the LBM simulations concurred with the experimental data before the stall condition, as shown in Fig. 5(a). The LBM prediction of the lift coefficient was under-predicted by approximately 10% at an angle of attack of 8° owing to the highly unsteady behavior resulting from the flow separation. In Fig. 5(b), the overall trend of the drag coefficients varying with

the angle of attack matches well with the measurements and other simulation results; however, the predicted values are under-estimated, which may be attributed to limitations in wall modeling and mesh resolution near complex ice geometries. The under-prediction of drag coefficients becomes more pronounced at higher angles of attack. This trend can be attributed to the flow separation occurring on the upper surface of the iced airfoil as the angle of attack increases, which significantly increases pressure drag due to adverse pressure gradient and separated flow. The complex nature of the separation and reattachment processes in the iced airfoil presents a considerable challenge, even for advanced computational methods. The comparative analysis showed that, the drag coefficients obtained by the RANS and ZDES simulations are also under-estimated, similar to LBM results. The RANS approach tends to underpredict both drag and lift under most conditions because it fails to accurately capture flow separation and recirculation. The ZDES method, although more capable of resolving large-scale unsteady structures, still relies on RANS-based near-wall models, which can limit its accuracy in regions involving complex flow separation. The LBM, being inherently transient and capable of resolving fine-scale flow features, may also exhibit discrepancies due to grid-resolution limitations and simplified near-wall modeling assumptions.

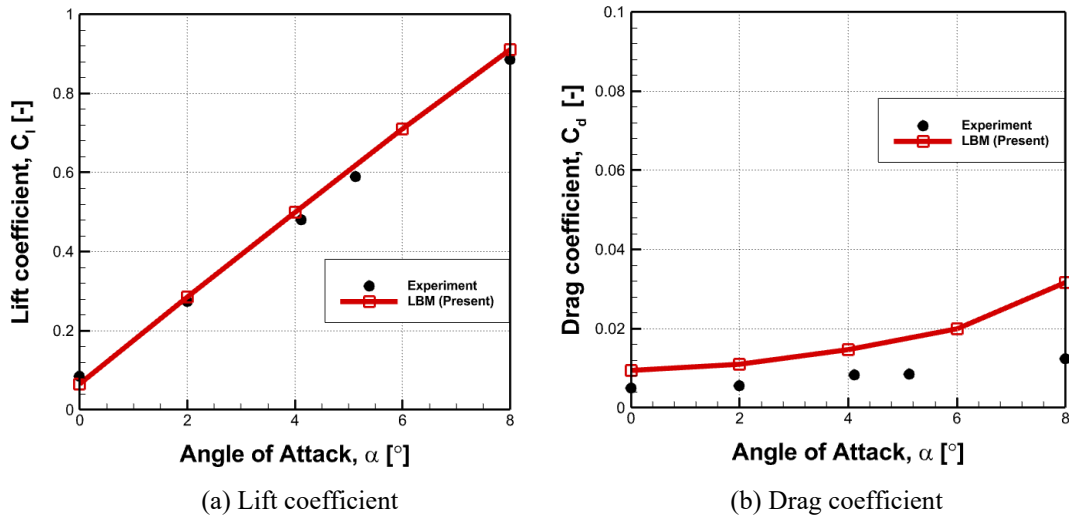


Fig. 4. Comparison of the aerodynamic coefficients of the clean airfoil with respect to the angle of attack.

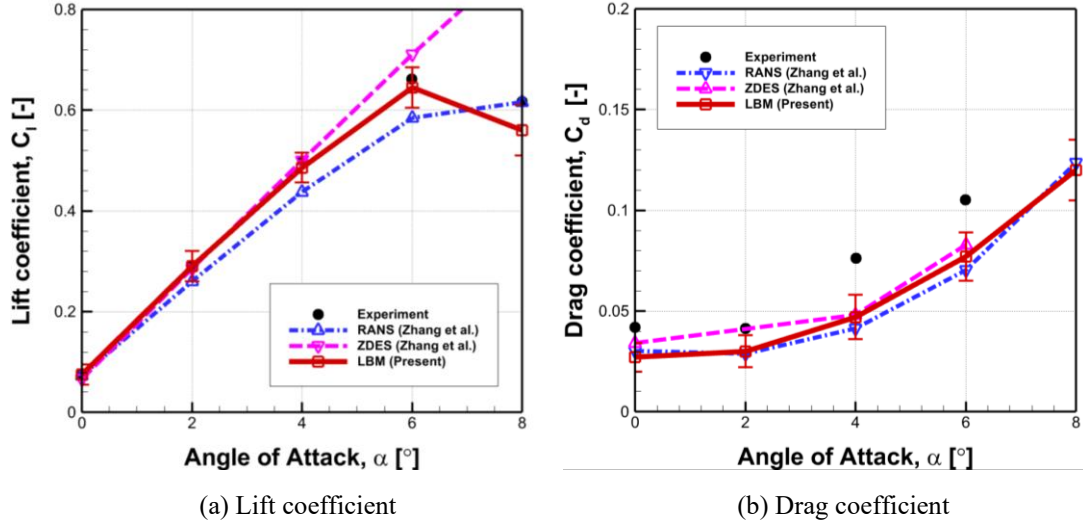
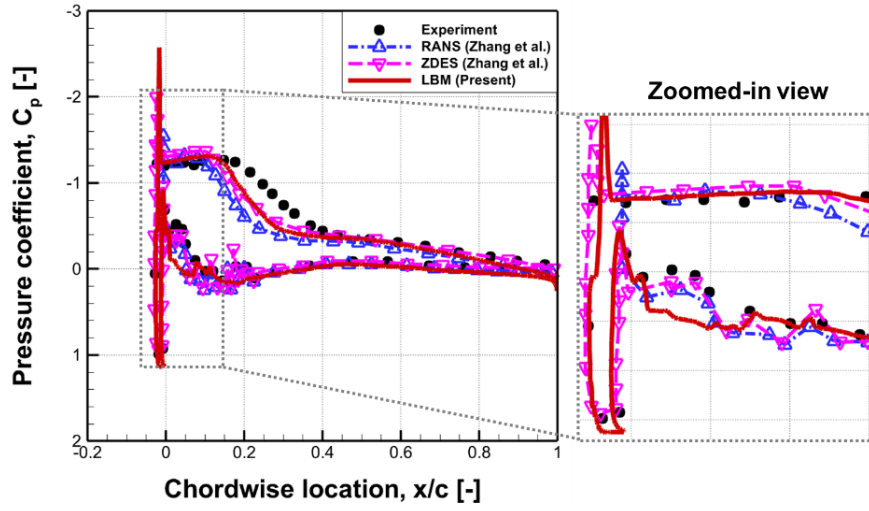


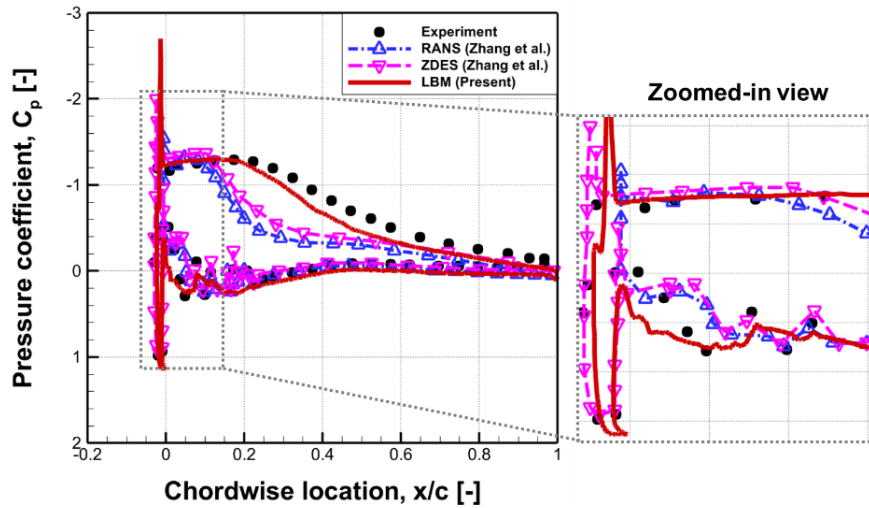
Fig. 5. Comparison of the aerodynamic coefficients of the iced airfoil with respect to the angle of attack.

Figs. 6(a) and 6(b) illustrate a comparison of the distribution of the averaged pressure coefficients of the iced airfoil at the angles of attack of 4° and 6° , respectively, considering the measurements and numerical results obtained from RANS, ZDES [58], and LBM simulations. The calculated pressure coefficient distribution on the suction and pressure sides of the iced airfoil appeared to be in good agreement with the experimental results, and the variation in the pressure coefficient, attributed to the ice accretion on the leading edge up to $x/c = 0.24$, was captured well. The pressure coefficient distribution along the suction surface between $x/c = 0.1$ and 0.4 was under-predicted at an angle of attack of 4° in comparison with the experimental results; however, the pressure distribution trend in the rest of the region matched well. Additionally, the pressure distribution was excellently predicted between $x/c = 0.1$ and 0.24 at the angle of attack of 6° . Although the surface pressure distributions from LBM simulations do not perfectly match with measurements, they show a closer match to experimental results than the RANS and ZDES models, particularly in Fig. 6(b). In particular, the ice-induced flow separation and reattachment, which dominate the surface-pressure behavior at an angle of attack of 6° , are more realistically captured within the LBM framework. The bounce-back wall model enables a physically consistent representation of near-wall flow behavior, while the kinetic-based LBM formulation naturally resolves nonlinear interactions within the separated boundary layer. Moreover, at this angle, the flow remains in a pre-stall condition, where the dynamics of separation and reattachment are strongly influenced by the local pressure gradient. The ZDES approach, although more advanced than RANS, still relies on model switching and grid-based criteria, which may limit its accuracy in predicting such complex unsteady features. Our findings indicate that LBM still offers a reasonable compromise between computational cost and accuracy, particularly when compared to RANS and ZDES methods. Over-predicted suction peaks were observed in both the LBM simulation and numerical analysis performed using

the RANS and ZDES methods [34, 58, 59], owing to the difficulty in measuring suction peaks of iced airfoils using pressure taps.



(b) Angle of attack of 4°

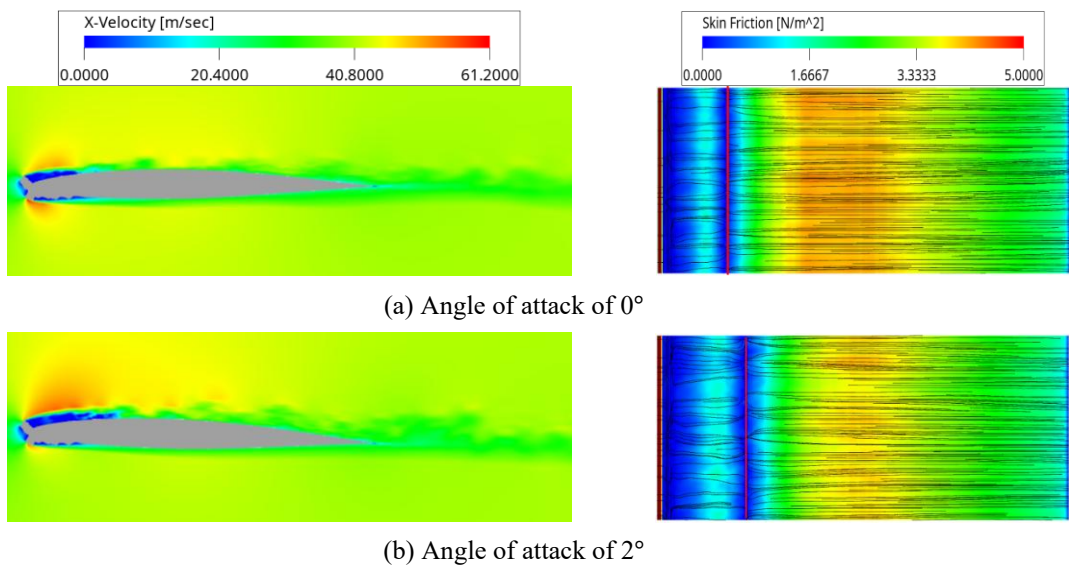


(b) Angle of attack of 6°

Fig. 6. Comparison of pressure coefficients of the iced airfoil at angles of attack of 4° and 6°.

Fig. 7 and Fig. 8 illustrates the instantaneous streamwise velocity and skin friction contours of the iced and clean airfoil at different angles of attack, which are indispensable for analyzing and understanding the impact of icing on the flow separation phenomena occurring in the airfoil. Ice accretion induced a separation bubble on the suction side of the iced airfoil, with the streamwise velocity turning zero. In contrast to the clean airfoil, it is worth noting that the leading-edge ice accretion with a dual-horn shape gives rise to notable flow separation phenomena on both the upper and lower surfaces of the iced airfoil. A modest separation angle was observed for angles of attack ranging from 0° to 4°, and the flow tended to attach to the upper surface of the iced airfoil along with a

pronounced reattachment force, thereby leading to the manifestation of a constrained region of flow separation. The ice-induced flow separation became more pronounced with the increase in the angle of attack, which in turn increased the size of the separation bubble and the distance at which reattachment occurred. The shear force and adverse pressure gradient responsible for flow separation in the airfoil gradually increased, triggering a transition in which the reattached flow near the trailing edge began circulating in the streamwise direction at the angle of attack of 6° within the pre-stall regime. The ice-induced flow separation remained unattached, and a highly unsteady and turbulent flow field developed on the upper surface of the iced airfoil at an angle of attack of 8° . As observed in streamwise velocity contours, the ice-induced separation and reattachment phenomena with varying angles of attack were further investigated using skin friction with streamlined contours. Similar to the points where the streamwise velocity reached zero in the contour, the location of the reattachment point (indicated by red lines on the contour) tended to move toward the trailing edge of the iced airfoil as the angle of attack increased. The entire flow on the upper surface of the iced airfoil was completely separated at an angle of attack of 8° , making it impossible to discern the reattachment region. Moreover, the magnitude of skin friction decreased with the increase in the angle of attack, reducing the force responsible for reattaching the flow to the surface. For the clean airfoil at angles of attack of 4° , 6° , and 8° , the flow remains largely attached over the entire chord. Although local variations in streamwise velocity are observed near the wall due to boundary-layer development, the temporal variation of the flow field is relatively small, and no large-scale separation or reattachment phenomena are detected. The skin-friction magnitude of the clean airfoil is consistently higher than that of the iced airfoil at the same angles of attack, indicating stronger wall shear and a greater resistance to flow separation.



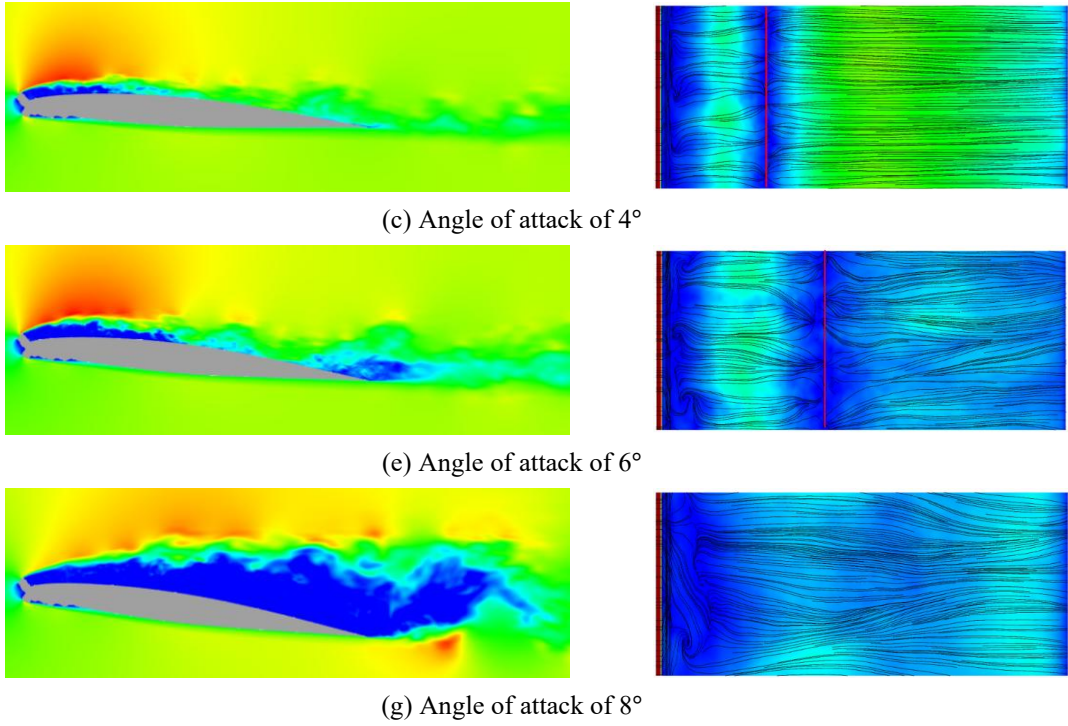


Fig. 7. Instantaneous streamwise velocity (left) and skin friction (right) contours of the iced airfoil at different angles of attack.

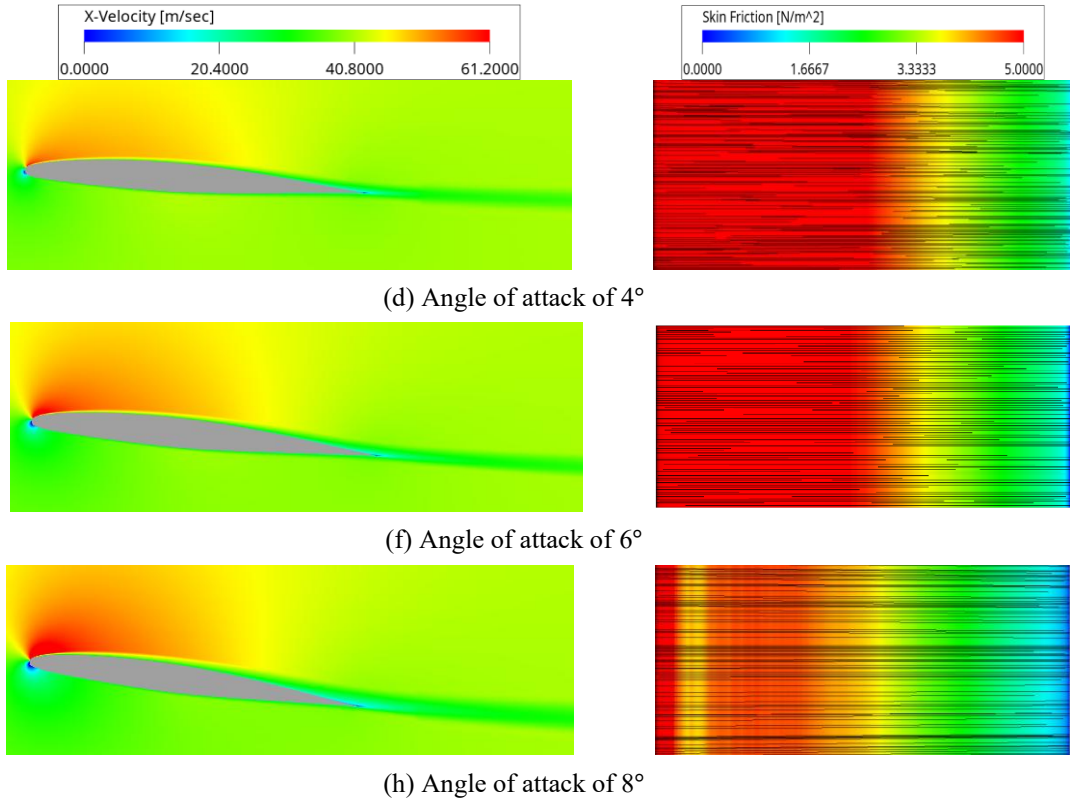


Fig. 8. Instantaneous streamwise velocity (left) and skin friction (right) contours of clean airfoil at different angles of attack.

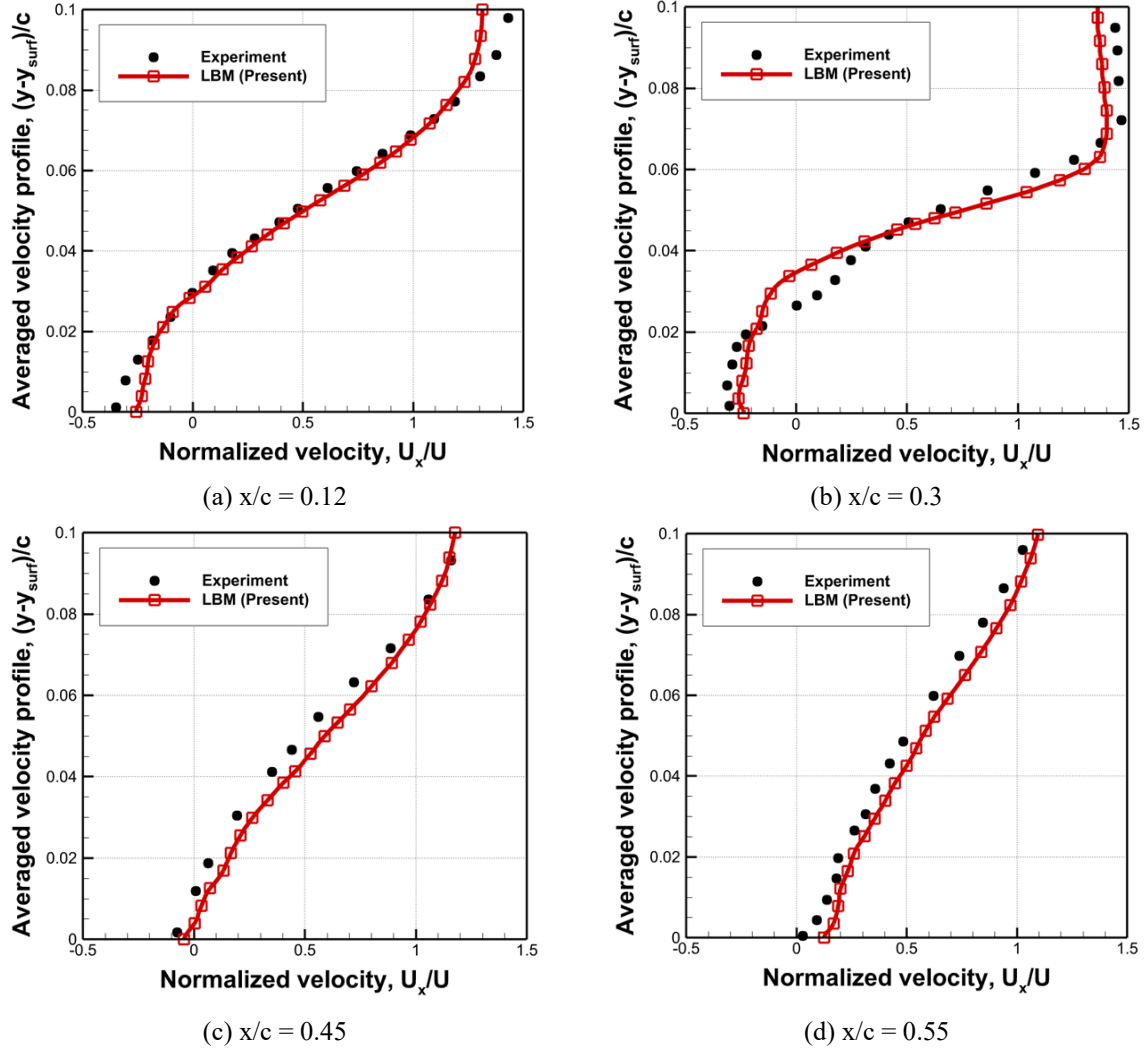


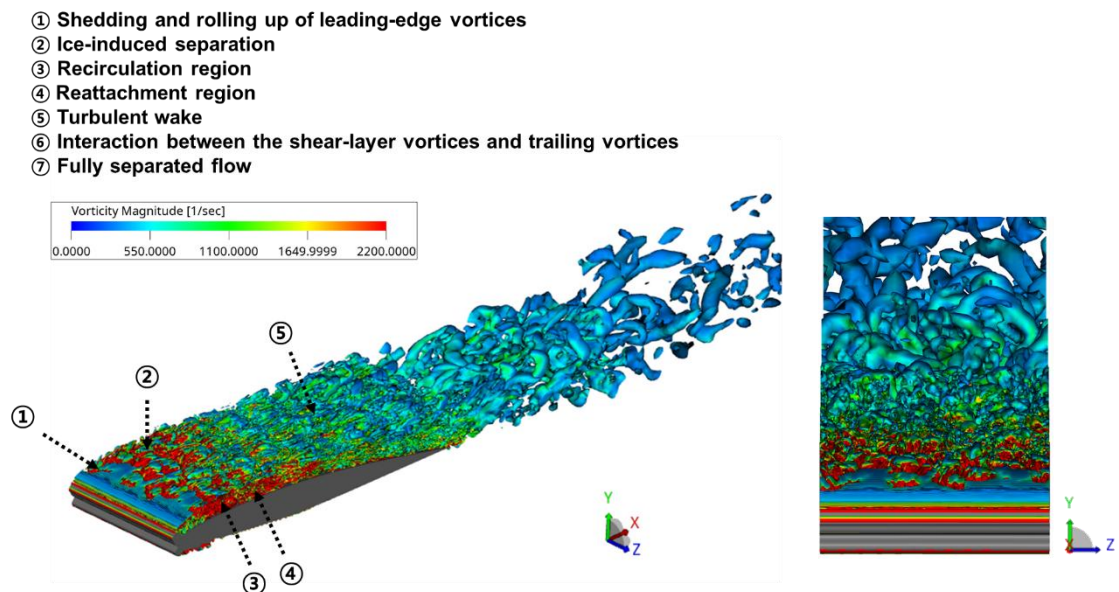
Fig. 9. Normalized velocity profile on the upper surface of the iced airfoil at an angle of attack of 6° at four positions along the streamwise direction.

Fig. 9 depicts a comparison of the mean normalized velocity profile in the turbulent flow field on the suction side of the iced airfoil at an angle of attack of 6° obtained using the LBM predictions and experimental results. A comparative analysis was performed for the solver validation at four specific locations along the streamwise direction: $x/c = 0.12, 0.3, 0.45$, and 0.55 , defined in Fig. 1. The LBM simulation yielded results that were notably consistent with the measurements at all four positions. At the first and second positions ($x/c = 0.12$ and 0.3), the velocity near the wall was in the direction opposite to the primary flow (positive x -direction) owing to their locations within the recirculation zone. Apart from the magnitudes of the normalized velocity components in the vicinity of the wall and outer boundary layer region being slightly underestimated, both the distribution and

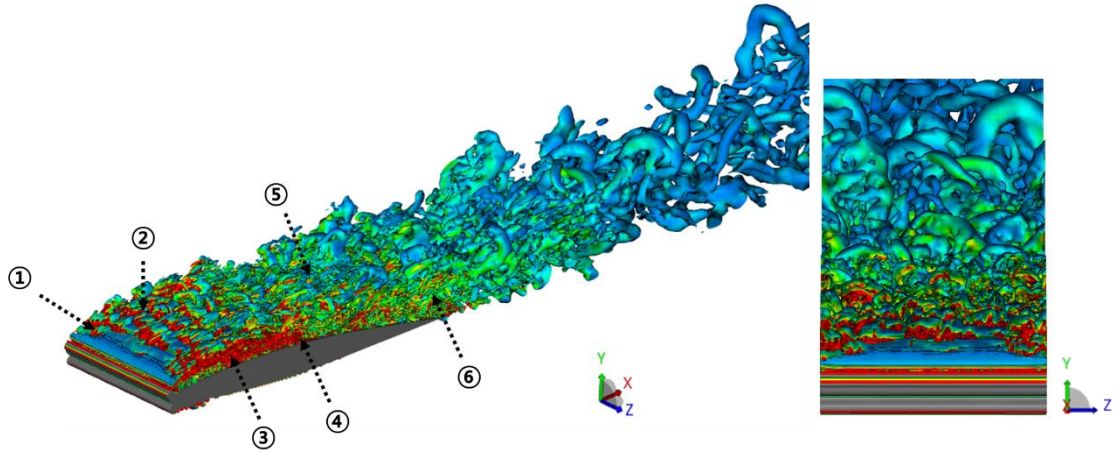
magnitude of the velocity profile matched the experimental data well. It found that the magnitudes of velocity near the wall approached zero at the third and fourth positions ($x/c = 0.45$ and 0.55), corresponding to the reattachment points. Similar to the previous observation, the velocity development along the vertical position was captured well, although the velocity component close to the wall was slightly underpredicted in comparison with the experimental results.

3.2 Turbulent Flow Field

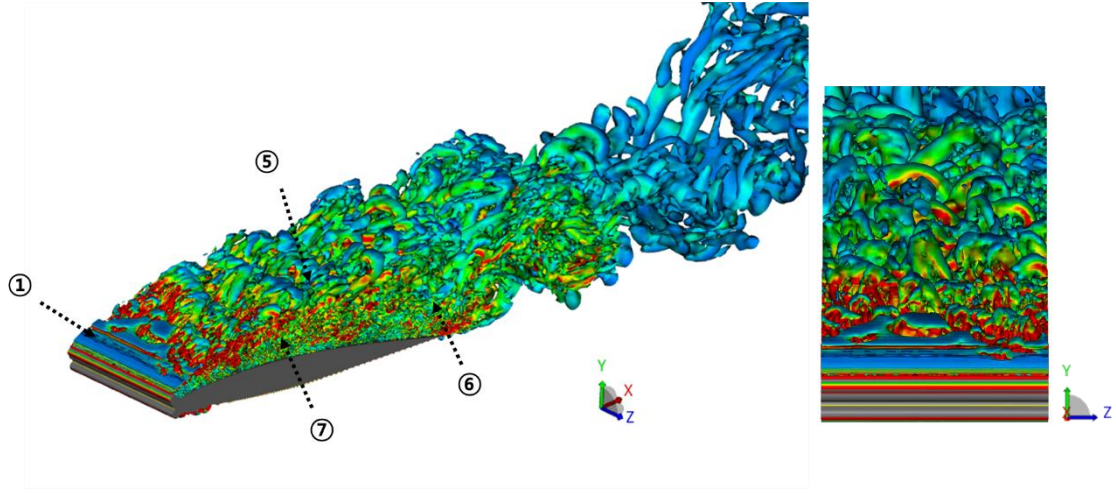
The instantaneous iso-surfaces of λ_2 -criterion colored by the vorticity magnitude over the iced airfoil at different angles of attack are depicted in Fig. 10. The development process of the ice-induced separation and wake structures with abundant multiscale turbulent features was captured. Although a clean-airfoil comparison is not presented in this study, the observed early separation and persistent vortex formation downstream of the leading-edge ice indicate a fundamentally different flow behavior compared to that of typical clean airfoils, where separation tends to develop more gradually near stall. These features are consistent with known characteristics of ice-induced separation reported in the literature [1,2]. As illustrated in Fig. 8, no significant disparity existed in the ice-induced separation phenomena, which encompassed the recirculation region and flow reattachment at the angles of attack ranging from 0° to 4° because the separated flow caused by the leading-edge ice accretion reattached to the upper surface of the iced airfoil. Therefore, the turbulent flow fields occurring around the iced airfoil were investigated at angles of attack of 4° , 6° , and 8° .



(a) Angle of attack of 4°



(b) Angle of attack of 6°



(c) Angle of attack of 8°

Fig. 10. Instantaneous iso-surfaces of the λ_2 -criterion colored by the vorticity magnitude.

An ice-induced separation bubble, which is the flow separation caused by the formation of ice on the leading edge of the airfoil, was observed at all angles of attack, as shown in Fig. 10. In Fig. 10(a) the flow passing through the ice accretion horn was no longer able to follow a region on the suction side of the airfoil, despite the angle of attack being significantly lower than the stall onset angle. The detachment of the boundary layer from the surface, induced by the double-horn-shaped ice accretion on the leading edge of the airfoil, led to the generation of leading-edge vortices via the shedding and rolling-up processes. Additionally, the detached flow turned into vortical structures, resulting in a turbulent wake behind the ice accretion. Fig. 10(b) illustrates the interaction between shear-layer vortices and trailing-edge vortices at an angle of attack of 6° . This interaction induces unsteady vortex dynamics, including vortex stretching, merging, and breakup, which are manifested as complex, time-varying structures in the instantaneous vorticity field. The flow remains largely attached beyond the mid-chord, suggesting

that the aerodynamic surface still supports partial pressure recovery and lift generation despite the disturbance caused by the leading-edge ice accretion. Fig. 10(c), in contrast, shows a marked weakening of vortex intensity and complete flow separation over the entire suction surface at an angle of attack of 8° , with no observable reattachment. The fully separated turbulent wake interacts with the trailing-edge vortices, forming coherent structures rich in multi-scale turbulence. Downstream of the separation point, the flow becomes highly disorganized and dominated by large-scale unsteady motions. This condition corresponds to the post-stall regime, where ice-induced aerodynamic degradation is most severe. Counter-rotating vortex pairs emerge in the turbulent wake; these vortices exhibit strong instability, oscillate periodically, and eventually shed downstream in a quasi-periodic swirling pattern.

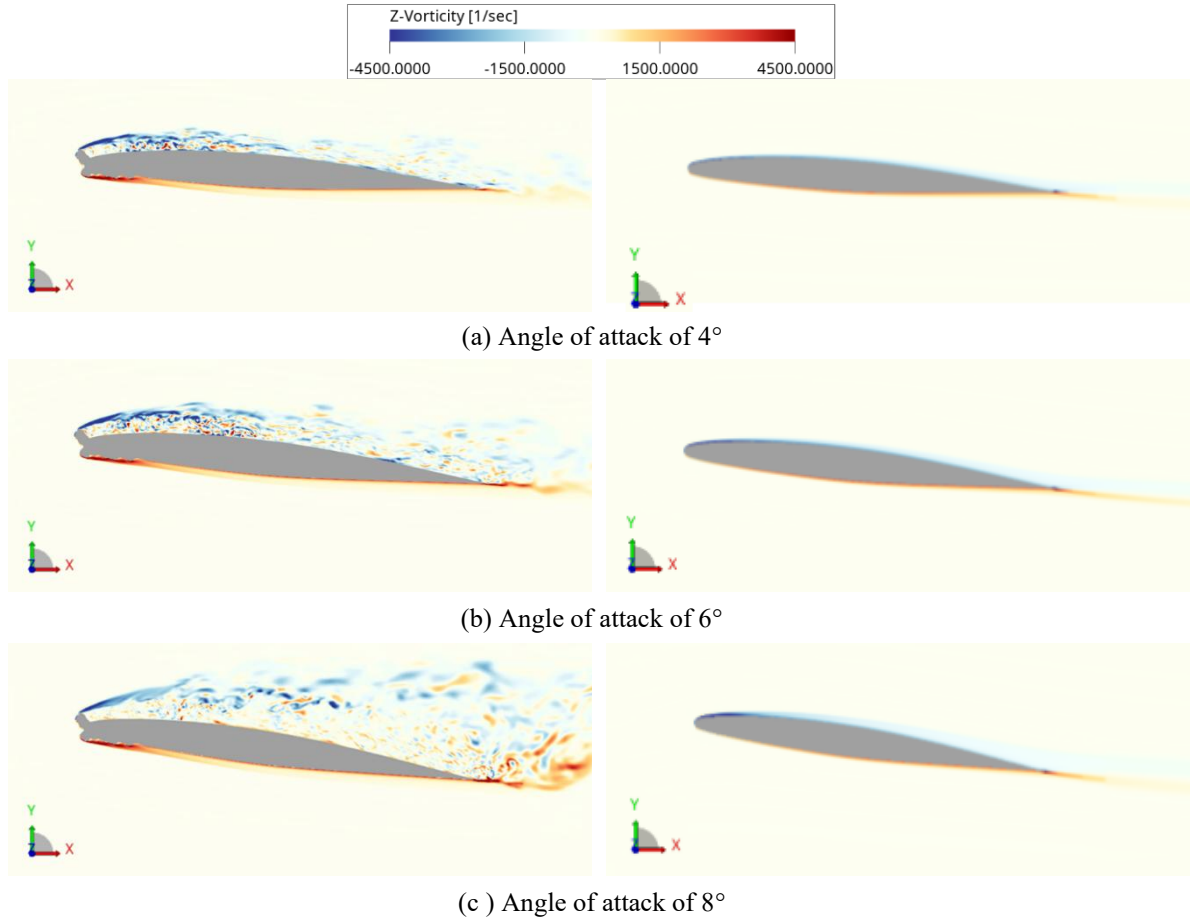


Fig. 11. Instantaneous spanwise vorticity contours of the iced airfoil (left) and clean airfoil (right) at different angles of attack.

In Fig. 11 depicts the instantaneous spanwise vorticity distribution of the transient flow field around the iced airfoil at the angles of attack of 4° , 6° , and 8° . Overall angle of attacks, the interaction of the unsteady separated flow over the iced airfoil moving along the streamwise direction with the external flow formed a recirculation

region containing strong vorticity prior to the reattachment location. The vortical structure of the wake of the iced airfoil became dynamically unstable and eventually dissipated as it propagated in a streamwise direction. In Fig. 11(a), the flow separates shortly after passing over the ice accretion, and a series of periodically spaced vortices with alternating signs form along the shear layer just downstream of the ice. These roll-up patterns clearly demonstrate the presence of Kelvin–Helmholtz (K–H) instability, which develops due to the strong velocity gradient between the high-speed external stream and the reverse flow inside the separation bubble. The alternating red and blue regions in the spanwise vorticity field represent counter-rotating vortices that roll up and evolve into coherent structures. As these vortices grow in strength, they promote entrainment between the outer flow and the separated region, causing the shear layer to roll up and eventually reattach to the suction surface. The reattachment forms a compact recirculation zone, after which the flow convects downstream toward the trailing edge while shedding residual vortices into the wake [60]. Fig. 11(b) presents a similar flow pattern at a higher angle of attack, but with a larger separation region and an increased vortex-shedding angle. As the angle of attack increases, the interaction between the shear-layer vortices and trailing-edge vortices intensifies, leading to secondary instabilities such as vortex pairing and deformation. These processes enhance the formation of unsteady turbulent structures in the downstream region. Furthermore, as larger amounts of separated flow reattached to the upper surface of the airfoil, the recirculation region expanded with the increasing vorticity inside the region. Fig. 11(c) shows that when the angle of attack increased to 8° , the shedding angle increased and flow separation occurred owing to the ice accretion attached to the leading edge. The same phenomena were observed at the angles of attack of 4° and 6° . However, the separated flow was unable to reattach to the upper surface and propagated downstream. The absence of reattachment and the long residence time of the separated flow enhance the vortex stretching and dissipation, as seen by the reduced coherence and diffuse vorticity field in the downstream direction. Eventually, the shedding and rolling up of the separated flow strongly interacted with the external flow, which enhanced the dissipation of vorticity, resulting in a relatively weaker vorticity distribution on the suction side of the iced airfoil. However, these vortices interacted strongly with the vortices generated at the trailing edge, leading to periodic shedding of the vortical structure of the turbulent wake in the downstream direction. Overall, the observations confirm that K–H instability plays a dominant role in triggering flow separation at lower angles of attack, while vortex interaction and deformation become the prevailing mechanisms at higher angles, particularly near stall. In contrast, the clean airfoil exhibits fundamentally different vorticity characteristics across all angles of attack. At angles of attack of 4° and 6° , the spanwise vorticity remains confined to a thin region near the wall, consistent with an attached boundary layer and the absence of large-scale flow separation. No coherent shear-layer roll-up

or K–H instability is observed. At an angle of attack of 8° , a slight increase in vorticity magnitude is observed along the suction side, however, this vorticity remains weak and spatially limited, indicating boundary-layer-related shear rather than separated-flow dynamics. Overall, the clean airfoil does not exhibit sustained vortex shedding or large-scale vortical structures, highlighting that the dominant source of unsteady vorticity in the iced configuration originates from ice-induced flow separation and shear-layer instability.

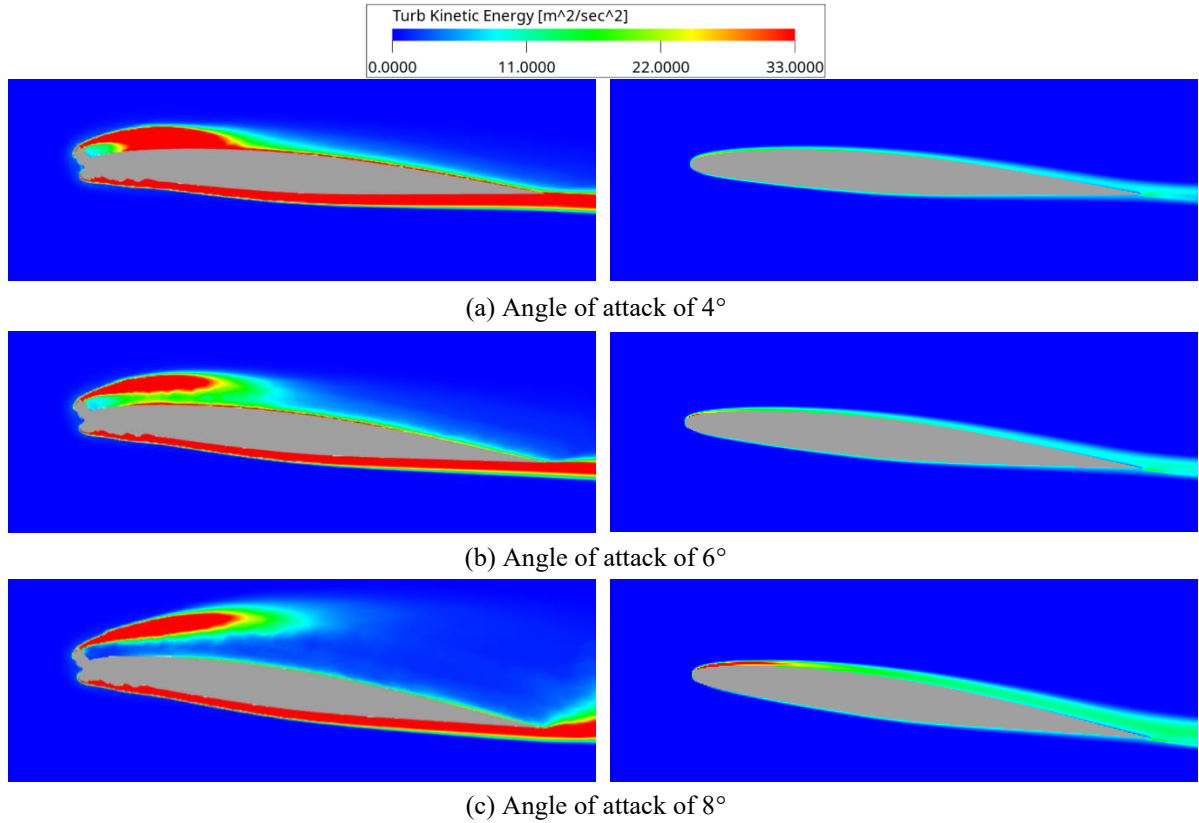


Fig. 12. Averaged turbulence kinetic energy contours of the iced (left) and clean airfoil (right) at different angles of attack.

Fig. 12 shows the averaged turbulence kinetic energy (TKE) contours at angles of attack of 4° , 6° , and 8° ; these contours facilitated the investigation of the turbulent flow around the iced airfoil. TKE is an indicator of the intensity of turbulence and understanding it is essential for analyzing noise caused by turbulence. The contour of the TKE on the iced airfoil represented the physical significance of turbulent energy levels, indicating regions of the airfoil where the turbulence was more intense or where the kinetic energy associated with turbulence was higher. Ice accretion on the leading edge of the airfoil substantially impacted the TKE distribution around the airfoil, and changes in the angle of attack exhibited a further impact on the TKE levels. Understanding the variations in TKE with changing angles of attack is crucial for assessing the impact of ice accretion on the

aerodynamic performance of iced airfoils and the corresponding noise generation. This provides valuable insights into the complex interactions between airflow, turbulence, and flow-induced noise. The TKE peaks (regions of maximum TKE) at each angle of attack were located immediately downstream of the upper ice-horn tip. The presence of ice resulted in an uneven TKE distribution along the upper and lower surfaces of the airfoil as shown in Fig. 12. At an angle of attack of 4° (Fig. 12(a)), the double-horn-shaped ice accretion on the leading edge of the airfoil disrupted the incoming flow despite the small angle of attack, introducing turbulence in the boundary layer. This disruption increased the TKE near the leading edge of the airfoil. The irregular shape of the ice accretion served as a trip point, initiating the turbulent flow earlier than it would have occurred on a clean airfoil. Moreover, a strong TKE was distributed near the wall because of the strong reattachment phenomenon at an angle of attack of 4° , similar to the results indicated in Fig. 10.

The variation in the angle of attack influenced the shear stress within the boundary layer. The shear stress became more pronounced as the angle increased, leading to increased TKE levels, particularly in regions where ice-induced flow separation and reattachment occurred. This is because a higher angle of attack results in a more dynamic and turbulent boundary layer. Consequently, areas of the airfoil affected by ice experienced a higher TKE owing to the disturbed flow and potential separation caused by the ice shape. At an angle of attack of 6° , the TKE levels near the wall gradually weakened with the increase in the angle of attack, and the TKE peaks moved away from the suction side as the angle of attack changed. At the angle of attack of 8° , the decrease in the TKE near the wall indicates the transformation of the ice-induced separation bubble structure to a stall, which was caused by the decrease in the reattachment force near the wall owing to the alleviation of the vorticity near the wall caused by ice accretion [61]. Moreover, an increase in the angle of attack resulted in a higher TKE near the trailing edge of the airfoil. This intensification was caused by the strong interaction of the ice-induced vortices with the vortices generated at the trailing edge, generating a more turbulent boundary, as we discussed in Fig. 11. In contrast, the clean airfoil exhibits markedly different TKE characteristics across all angles of attack. At angles of attack of 4° and 6° , elevated TKE remains confined to a thin region adjacent to the wall and is primarily associated with boundary-layer shear. No significant TKE amplification is observed over the suction side, and the flow remains largely attached without evidence of separation-induced turbulence. Even at an angle of attack of 8° , the clean airfoil shows only a moderate increase in TKE near the wall, while the outer flow remains largely quiescent. The absence of large-scale flow separation and shear-layer instabilities prevents the formation of high-TKE regions away from the surface, indicating that turbulence generation in the clean configuration is governed predominantly by boundary-layer dynamics rather than separation-driven vortical structures.

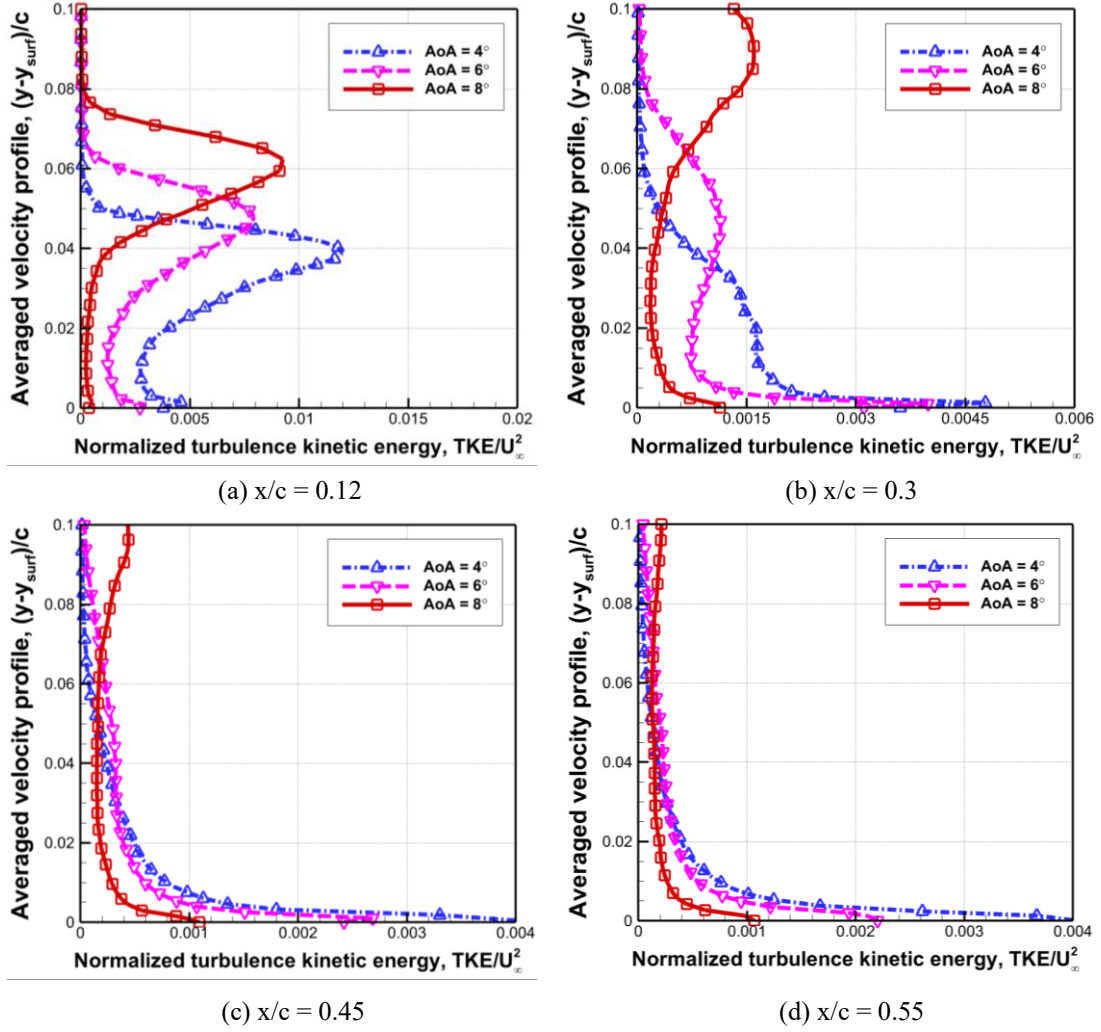


Fig. 13. Normalized turbulence kinetic energy profile with respect to the streamwise direction.

Fig. 13 illustrates the normalized TKE profile with respect to the streamwise direction at the four positions indicated in Fig. 1 ($x/c = 0.12, 0.3, 0.45$, and 0.55) on the suction surface for the angles of attack of $4^\circ, 6^\circ$, and 8° . At the first position ($x/c = 0.12$), which represents the area immediately after the ice shape, all the angles of attack comprised a recirculation zone sensitive to pressure and flow changes, and the maximum TKE occurred away from the wall owing to the stall [62]. Except for the significant turbulence generated at the angle of attack of 8° , the maximum TKE value increased with the increase in the angle of attack, and the boundary layer thickness decreased closer to the wall. At the location of $x/c = 0.3$, the overall size continued to reduce because of the ice shape; however, a high TKE value was observed away from the wall. At the angle of attack of 8° , the maximum TKE value occurred away from the wall, yet the TKE value near the wall decreased significantly compared with that observed at the angles of attack of 4° and 6° , which involved smaller recirculation zones. At the location of $x/c = 0.45$, which is the area where most of the flow reattached after recirculation, the turbulence generated after

the ice shape gradually decreased. Therefore, the TKE value decreased in all regions except for the area close to the wall, and the boundary layer thickness significantly reduced in all cases, except at the angle of attack of 8° . At the angle of attack of 6° , the maximum TKE value near the wall decreased compared to that at the angle of attack of 4° , owing to a decrease in the reattachment strength. At the location of $x/c = 0.55$, which is the area where the flow reattached after recirculation, the angle of attack of 4° exhibited a maximum TKE value near the wall, similar to that observed at the location of $x/c = 0.45$; however, the TKE near the wall decreased as the peak turbulence shifted into the separated shear layer. This is due to the weakening of near-wall reattachment and the upward displacement of turbulent structures. The TKE value decreased at the angle of attack of 8° , except in the area near the wall, owing to vorticity dissipation.

3.3 Far-field Noise

An aeroacoustic analysis of the iced airfoil was conducted to investigate the effect of the icing phenomenon on the characteristics of the radiated noise and its directivity at different angles of attack. Typically, ice accretion on the leading edge of an airfoil disrupts the smooth freestream, leading to premature flow separation and the formation of a turbulent region as air flows over an irregular ice surface. These turbulent regions significantly contribute to noise generation. The turbulent flow caused by ice accretion results in pressure fluctuations on the airfoil surface, which are accompanied by vortices that can induce unsteady fluctuations in air pressure and contribute to tonal noise production. Moreover, the presence of irregularly shaped ice leads to the shedding of vortices, generating discrete tones or tones at specific frequencies that can add to the overall noise produced by the iced airfoil. Furthermore, ice-induced turbulence and irregular flow patterns result in broadband noise that covers a wide range of frequencies and is typically associated with turbulent flows and pressure fluctuations. In this study, the FW-H acoustic analogy with permeable surfaces was used for predicting the far-field noise based on the results of the LBM simulations, as this method can effectively consider all noise sources in the iced airfoil. The prediction of far-field noise can be valuable for the development of ice detectors that employ acoustic signals. Fig. 14 shows the locations of the microphones around the iced airfoil and the size of the permeable surface enclosing the iced airfoil with complex turbulent flow structures. The microphones were placed at a distance of 10 times the chord length of the airfoil ($R = 10c$; here c denotes the chord length of the airfoil) with 10° intervals along the azimuthal direction (θ) respect to angle of attack 0° . In this study, the noise analysis was conducted for different angles of attack while keeping the microphone position fixed to investigate the effect of angle of attack changes on the aerodynamic and acoustic characteristics of the iced airfoil. By maintaining a constant microphone

position, we were able to focus on how variations in the angle of attack influenced the flow structures and aerodynamic forces, which in turn affect noise generation.

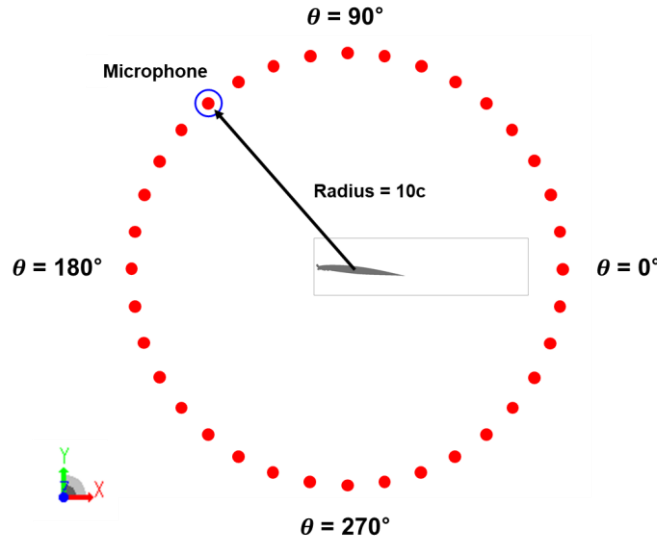
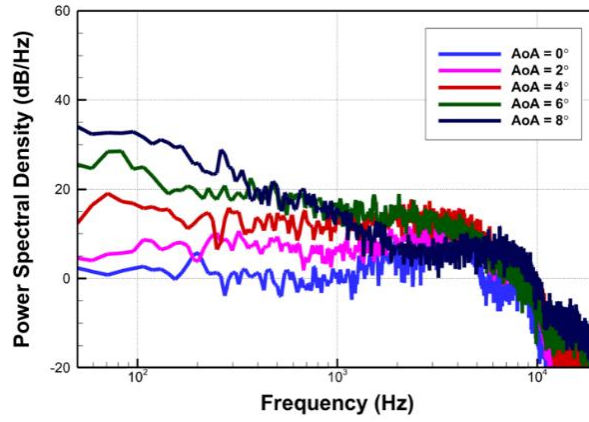


Fig. 14. Positions of microphones around the iced airfoil (not drawn to scale).

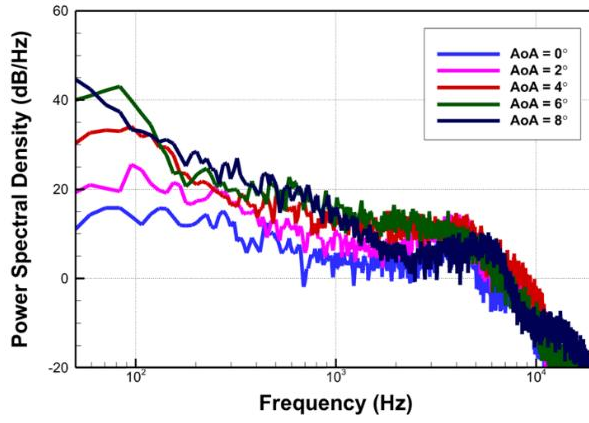
Fig. 15 presents the power spectral density (PSD), which is a frequency-domain plot of power per Hertz with respect to the frequency, for the iced airfoils within the range of the angles of attack from 0° to 8° for three azimuthal locations of 0° , 30° , and 90° . In our analysis, the cut-off frequency refers to the frequency threshold used to filter out noise is 50 Hz. The frequency range for the acoustic spectrum is 50-20,000 Hz. An increase in noise levels in the low-to-mid frequency range was observed owing to the ice-induced separation bubble generated by the formation of ice and the thick boundary layer [25]. When the airfoil was subjected to loading noise, the dominant noise was observed in the low-frequency range. It was found that the noise level increased in the low-frequency range as the angle of attack increased. However, at the angle of attack of 8° , the recirculation zone was not formed owing to the strong flow separation, and the overall vorticity magnitude decreased, which in turn reduced the noise level in the mid-frequency range before increasing it again in the broadband frequency range.

At an azimuth angle of 0° , which corresponds to the downstream direction, the noise level in the low-frequency range increases at angle of attack of 8° due to enhanced vortex shedding caused by fully separated flow. The complete breakdown of the recirculation bubble results in stronger, large-scale unsteady structures propagating downstream. In contrast, at lateral or upward directions (e.g., 30° , 90°), the noise sources, especially those related to turbulent separation bubbles, are weakened due to the absence of reattachment, resulting in lower acoustic radiation. The results further validated that the noise level increased with the azimuth angle, with the highest noise

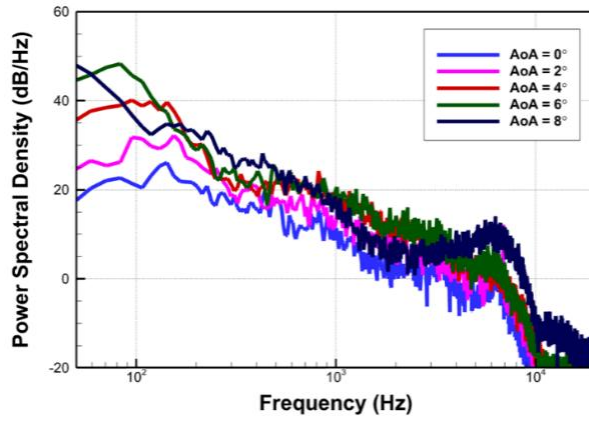
level at $= 90^\circ$, which was perpendicular to the flow direction. This was attributed to the contribution of unsteady loading noise modeled as a dipole source.



(a) Azimuth angle of 0°



(b) Azimuth angle of 30°



(c) Azimuth angle of 90°

Fig. 15. Power spectral density (PSD) of the iced airfoils at different angles of attack.

Fig. 16 depicts the comparison of the overall sound pressure level (OASPL) directivity of the iced airfoils at the different angles of attack ranging from 0° to 8° . Changes in the angle of attack affected the overall aerodynamic characteristics of the iced airfoil, including noise generation. In this study, the values of OASPL were evaluated at frequencies ranging from 50 Hz to 10,000 Hz, including low-frequency components related to large-scale flow structures and high-frequency components related to small-scale turbulence. However, as the angle of attack increases, the flow over the iced airfoil tends to separate from the accumulated ice and suction side, which leads to the unsteady turbulence flow structures with different scales. The practical way to obtain the detailed acoustic spectrum is to achieve a fine frequency resolution. In this study, the acoustic data in time domain for a duration of 0.15 s with a time step of 2.28×10^{-6} s was utilized for the acoustic analysis, which provided a frequency resolution of 7.38 Hz. However, due to the grid resolution and numerical dissipation in the LBM solver, the acoustically reliable frequency range of the present simulation extends up to approximately 20 kHz, as confirmed by the power spectral density results in Fig. 15. Although the spectral levels rapidly beyond this frequency, this range can cover the broadband aerodynamic noise components relevant to the iced-airfoil configuration. It can be seen that the symmetrical OASPL distributions in both the upper and lower directions are observed at all angles of attack, with the maximum OASPL occurring at the azimuth angles of 90° and 270° . This was because the loading noise with the dipole radiation pattern was the dominant noise in the iced airfoil. Furthermore, it is noted that a kidney-shaped radiation pattern directed toward the azimuth angle of 180° was observed at low angles of attack where the ice shape was present. An increase in the angle of attack intensified the turbulence and pressure fluctuations on the airfoil surface, leading to higher noise levels that triggered flow separation, particularly at higher angles. Flow separation is a significant contributor to airfoil noise because it creates vortices and turbulent flow regions that generate overall noise, including low-frequency peak noise. However, the increase in the noise level for the angle of attack of 8° was not significant compared to that for the angle of attack of 6° due to the decrease in the noise level in the mid-frequency range, as discussed in Fig. 15.

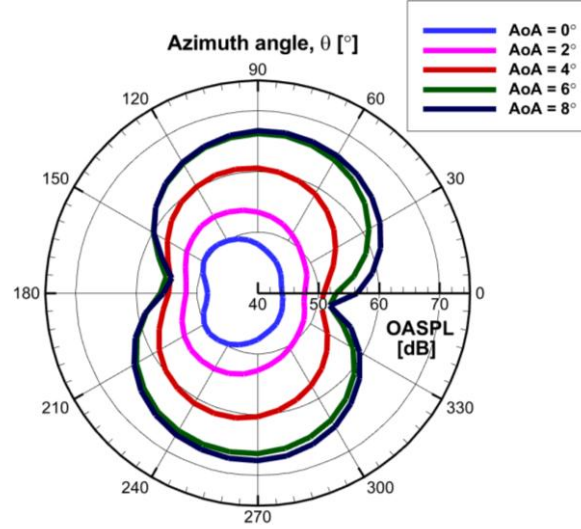


Fig. 16 Comparison of the overall sound pressure level (OASPL) directivity of the iced airfoil at different angles of attack.

3.4 Acoustic Characteristics

The time-derivative pressure contours around the iced airfoil at angles of attack of 6° and 8° are presented in Fig. 17 to identify the primary sources of acoustic waves and analyze their propagation in terms of direction and speed. We observed the changes in pressure over time at different points in a specific area to identify the locations where the sound waves originated or were the most intense. This information was crucial for understanding the underlying noise-generation mechanisms. Owing to the presence of ice accretion on the leading edge of the airfoil, flow separation was more likely to occur from an iced airfoil surface than from a clean airfoil. This flow separation created low-pressure regions and complex turbulent structures on the upper surface of the airfoil. Consequently, the time-derivative pressure contours indicated distinct patterns of low-pressure regions corresponding to the separated flow areas. Periodic pressure fluctuation was observed at angles of attack of 6° and 8° after passing the flow through the iced shape, and the magnitude of the pressure fluctuation intensified with an increasing angle of attack. The pressure changes over time were significant only near the suction side of the iced airfoil at an angle of attack of 6° because the ice-induced separation flow interaction with the trailing edge was not significant. Higher angles of attack tended to intensify turbulence, resulting in more pronounced pressure fluctuations. These fluctuations manifested as variations in the time-derivative pressure contours, leading to sharper peaks and deeper troughs. Moreover, significant pressure fluctuations along the airfoil occurred both in the vicinity of the ice shape and near the trailing edge at the angle of attack of 8° because of the strong interaction between the ice-induced vortices and vortices generated from the trailing edge.

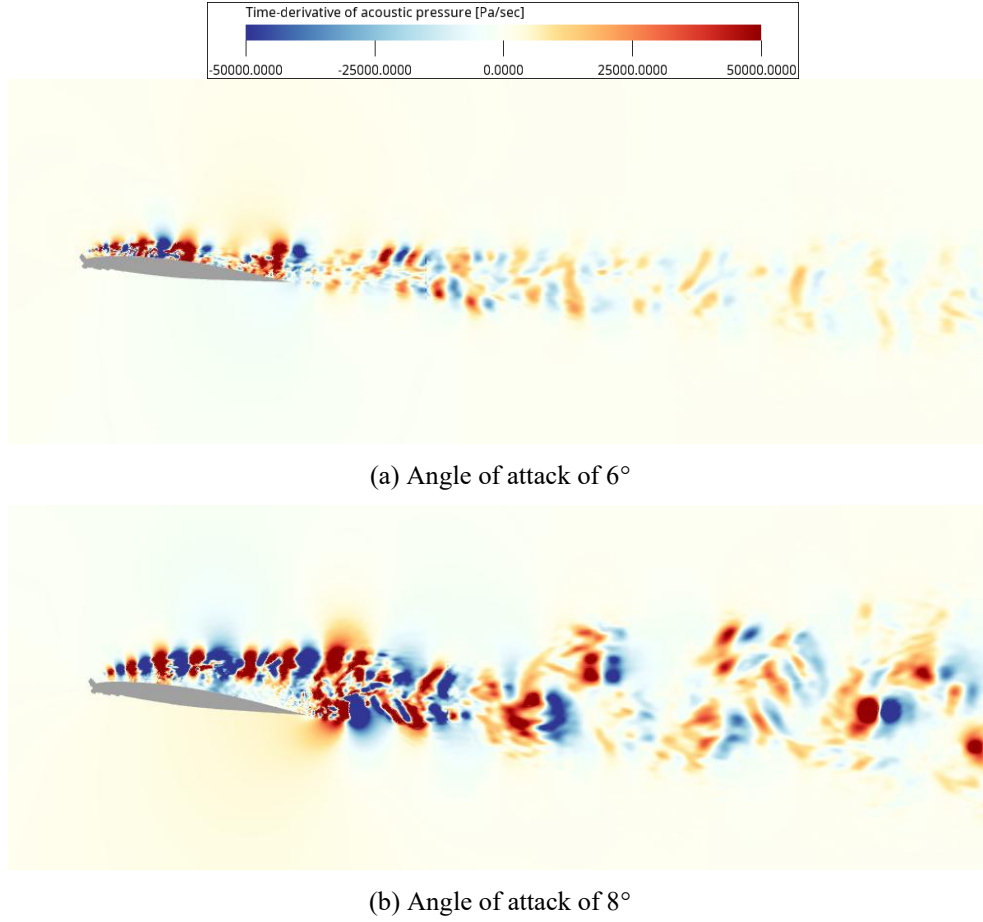
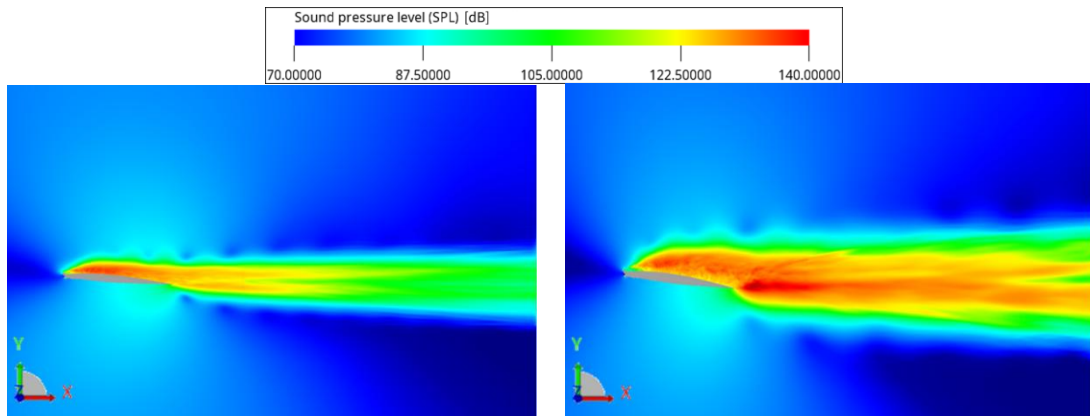


Fig. 17. Time-derivative pressure contours around the iced airfoil (left) and clean airfoil (right) at angles of attack of 6° and 8°.

The detailed frequency analysis was performed by plotting sound pressure level (SPL) contours in one-third octave bands. One-third octave band SPL contours around the iced airfoil at angles of attack of 6° and 8° are depicted in Fig. 18, respectively, representing the distribution of SPL across a range of specific frequency bands. The spectra are computed at all points on the contour, and the SPL contour between 70 dB and 140 dB is shown to visually represent the spectral characteristics of noise. One-third octave band analysis helps examine the spectral composition of noise generated by the iced airfoil and identify the dominant frequency components that contribute significantly to the overall noise levels; higher SPLs in a specific band indicate that more acoustic energy is concentrated within that frequency range. Although the magnitudes of SPL are similar in all the frequency bands, noise contribution in the low-frequency band is apparent for both the angles of attack of 6° and 8°. In the 100 Hz one-third-octave band, the SPL contours are primarily concentrated along the suction side from the mid-chord to the trailing edge. These localized high-SPL regions spatially coincide with the ice-induced separated shear layer, where large-scale vortex shedding and unsteady flow separation occur due to the horn-shaped ice accretion. The resulting coherent vortical structures interact with the trailing edge, generating

pronounced low-frequency pressure fluctuations. At an angle of attack of 8° , the high-SPL region becomes broader and extends further downstream, which is consistent with the enlargement of the separated region and the presence of more energetic, large-scale vortex shedding observed in the instantaneous flow field. At 160 Hz, the SPL distribution exhibits a similar spatial pattern but with increased intensity. This amplification is associated with the development of shear-layer instabilities within the separated flow, where strong velocity gradients promote the growth and interaction of large-scale coherent vortices. The interaction between the unstable separated shear layer and the trailing-edge flow enhances downstream-directed acoustic radiation due to the increased coherence and interaction of pressure fluctuations. In contrast, at 250 Hz, the SPL field becomes more spatially diffused and the overall intensity decreases. This behavior indicates a transition in the dominant noise-generation mechanism from large-scale coherent structures to smaller-scale turbulent fluctuations within the separated flow and wake region. Owing to the reduced coherence and shorter correlation length of these small-scale turbulent structures, the associated acoustic radiation exhibits weaker directivity and contributes to the far-field noise in a more isotropic manner. Overall, the low-frequency bands exhibit more dominant acoustic contributions than the higher-frequency bands, reflecting the strong influence of coherent separation-induced flow structures. In addition, as the angle of attack increases, the regions of maximum SPL shift closer to the trailing edge and expand in size, which is consistent with the intensification of turbulent flow behavior and the downstream migration of the separation region, as shown in Fig. 18.



(a) 100 Hz (band 89.125 Hz–112.202 Hz)

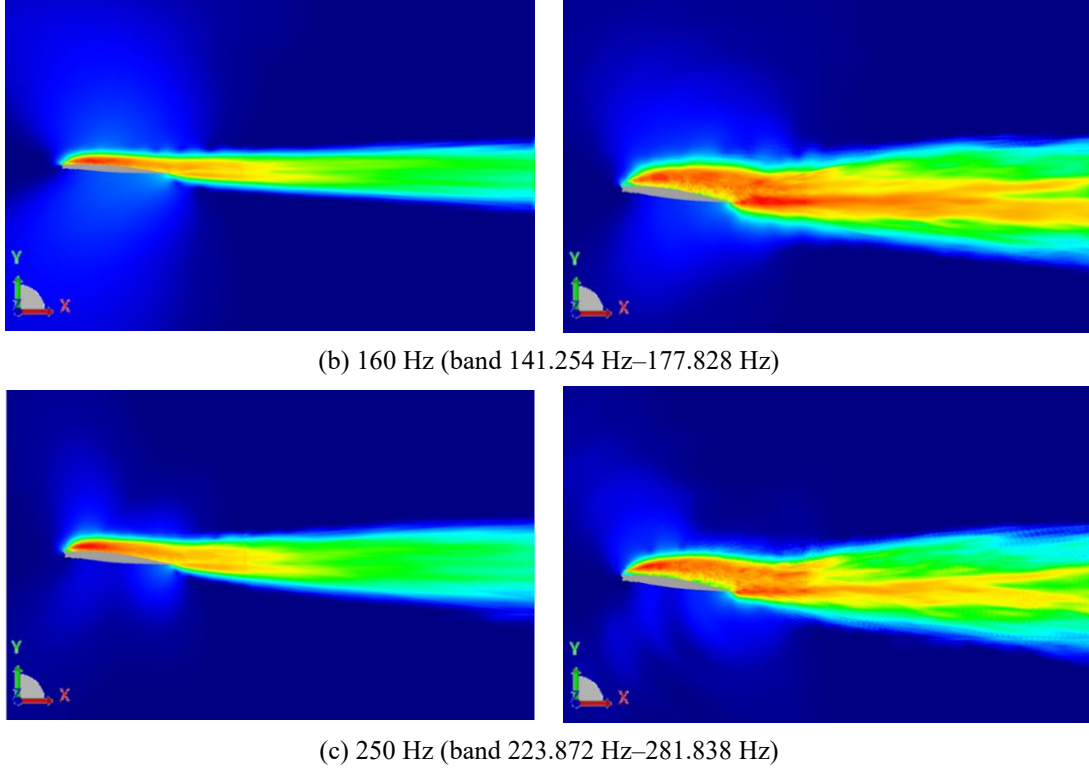


Fig. 18. One-third octave band sound pressure level (SPL; dB) contour of the iced airfoil: angles of attack of 6° (left) and 8° (right)

4. Conclusions

In this study, we performed an aerodynamic analysis of iced airfoils using the commercial software PowerFLOW based on LBM simulations and conducted noise analysis using the permeable FW-H formulation. The simulation results were validated against experimental data for an iced GLC305 airfoil tested at NASA IRT. Comparisons of lift coefficient, surface pressure, and velocity profiles across various angles of attack confirmed good agreement with the experimental measurements. Prior to the noise analysis, the unsteady flow field revealed the development of complex turbulent features such as Kelvin–Helmholtz instabilities, recirculation zones, and boundary layer separation behind the ice shape. The turbulent kinetic energy (TKE) analysis indicated the formation of separation bubbles and thickening of the boundary layer, especially under post-stall conditions, with peak TKE values occurring away from the wall. These unsteady structures were effectively captured by the LBM solver, providing high-fidelity input for acoustic prediction. Based on the aerodynamic analysis results, the far-field noise radiation characteristics of the iced airfoil were analyzed using the permeable surface-based FW-H acoustic analogy. Far-field noise was predicted using the permeable FW-H method, and results showed increased sound pressure levels in the low-to-mid frequency ranges. This increase was primarily attributed to the strong

turbulent separation bubbles and thick boundary layers generated by the ice shape. Time-derivative pressure contours and one-third octave band SPL plots identified dominant acoustic source regions downstream of the ice. Based on the results from aeroacoustic analysis, microphones placed at various azimuthal angles revealed a dipole-like noise radiation pattern. At higher angles of attack, noise was observed to propagate more toward the direction of the ice accretion. Based on this directionality, it is recommended to install acoustic sensors near the leading edge, where icing is most likely to form. Additionally, the findings suggest that acoustic sensors should be designed with enhanced sensitivity in the low-to-mid frequency range to improve icing detection reliability.

Overall, the results demonstrate that LBM-based approaches are effective for simulating unsteady flow and aeroacoustic phenomena in iced airfoils. Although the present study focused on two-dimensional configurations, future work will extend the analysis to three-dimensional wings and rotating blades. Parametric studies incorporating variables such as velocity, temperature, liquid water content, and mean volume diameter are also planned to further understand the aerodynamic and acoustic impacts of icing.

Declaration of competing for interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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