

**A Thesis for the Degree of Doctor of Philosophy**

**Computational Modeling of In-Flight Ice Accretion  
and Shedding on Rotorcraft using Hybrid  
Lagrangian-Eulerian Framework**

**By**

**Bidesh Sengupta**

**Department of Mechanical and Aerospace Engineering**

**Graduate School**

**Gyeongsang National University**

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**Computational Modeling of In-Flight Ice  
Accretion and Shedding on Rotorcraft using  
Hybrid Lagrangian-Eulerian Framework**

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Bidesh Sengupta**

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**Prof. Rho Shin Myong, Dissertation Supervisor**

**Prof. Hakjin Lee, Dissertation Co-supervisor**

**Approved by committees of the Graduate School of  
Gyeongsang National University in partial fulfilment of the  
requirements for the degree of Doctor of Philosophy**

**Dissertation Committee:**

Prof. Hakjin Lee

Chairman

Prof. Dr. Jae Hyun Park

Dr. Ji Hong Kim

Prof. Prince Raj Lawrence Raj

Prof. Rho Shin Myong

Supervisor

**Date: 2022.07**

**Department of Mechanical and Aerospace Engineering  
Graduate School  
Gyeongsang National University**

# **Acknowledgment**

To whoever I have interacted with in my life.

**BIDESH SENGUPTA**  
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# **Abstract**

## **Computational Modeling of In-Flight Ice Accretion and Shedding on Rotorcraft using Hybrid Lagrangian-Eulerian Framework**

Bidesh Sengupta

Department of Mechanical and Aerospace Engineering

Graduate School, Gyeongsang National University

Supervised by Prof. Rho Shin Myong

Co-supervised by Prof. Hakjin Lee

In-flight icing is a critical threat to the safety of a rotorcraft flying inside a cloud with supercooled droplets. Ice accumulations on the surface of rotorcraft significantly affect the performance of the rotorcraft. Hence, the design of a proper ice protection system can be a feasible option to prevent the adverse effects of ice accretion. For a proper ice protection system design, the aerodynamic, droplet impingement, and ice accretion behavior, along with ice shedding characteristics, need to be mastered.

In this study, a hybrid aerodynamic solver coupling Lagrangian and Eulerian framework to define wake and flow field around fuselage respectively was formulated to predict effectively and efficiently flow field around fuselage at any arbitrary advance ratio considering rotor wake effects. Next, an Eulerian-based droplet impingement code which provides the collection efficiency for air flows around any three-dimensional model containing water droplets is developed. A Finite Volume Method (FVM) is used to solve shallow water-based droplet equations. An unified grid technology is developed to use the same grid in both air and droplet solvers. A PDE-based ice accretion solver is also developed to predict the iced shape



on the clean geometry. The ice accretion solver is able to handle all regimes of ice formation in which the water film flow is automatically solved along with the mass of ice and equilibrium temperature. Further, the ice accretion solver is based on FVM, in which the inputs can be easily given from the air and droplet solvers.

The solvers were utilized to predict ice accretion around the ROBIN rotorcraft. An aerodynamic solver was utilized to predict the flow field around the fuselage. After the convergence of the rotor thrust coefficient, the flow field was averaged for one revolution and was used to initialize the droplet solver. The droplet solver calculated collection efficiency on the surface of the fuselage, and then an ice solver was employed to predict ice accretion location and mass. The two parameters, namely, advance ratio and rotor thrust coefficient, were varied in different combinations to study ice accretion, and it was found that the advance ratio had a dominant effect on the ice accretion as compared to the rotor thrust coefficient.

Ice shedding is a critical issue in the rotorcraft system and is a safety concern. Interaction of ice fragments with engine intake and tail rotor may damage the compressor blades and tail rotor, respectively, causing incidents and accidents to the rotorcraft. The present work established a general framework to study the ice shedding problem by including 6-DoF, Artificial Neural Network (ANN), and Monte Carlo (MC) to obtain ice footprint maps on the plane of critical components. In summary, the present work with the creation of several modules provides a concise platform to study ice accretion and ice shedding in a rotorcraft flow field with low computational resources and time with adequate accuracy.