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Analytical results on MHD intermediate shocks

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Abstract. The admissibility of MHD intermediate shocks has been a matter of much debate over the years. Though the legitimacy of such shocks was shown in a recent series of investigations by Wu, rigorous analytical proof of the results was considered being unsettled. In this paper, as a step towards developing such proof, we present a theory of MHD shocks. On the basis of global analysis, we develop a shock admissibility condition. Using this theory, we explain how intermediate shocks and compound waves are generated.

Introduction

The nature of shock waves is of importance to the understanding of physical phenomena arising in various mediums. Since shock waves are omnipresent in space plasmas, for example, magnetic field reconnection [*Lin & Lee*, 1994], they have been studied with great intensity. However, the mathematical analysis involves awkward algebra, so that their exact properties are not well understood.

In the past, the evolutionary theory was used to select physically relevant shocks [*Akhiezer et al.*, 1959]. According to this theory, intermediate shocks, which can be defined as shocks that lead to a transition from super-Alfvénic to sub-Alfvénic flow, are considered inadmissible. In the framework of this theory, the rotation of transverse magnetic field is achieved only by the rotational discontinuity.

Contrary to this theory, it was shown by the pioneering numerical experiments of Wu (1987) that some intermediate shocks can exist, whereas the rotational discontinuity cannot exist. A similar conclusion was drawn in the study of evolutionary conditions of intermediate shocks by Hada (1994). Wu (1995) also found that the evolution and structure of intermediate shocks are related, so that the global solutions can be affected by the local structure. Furthermore, the structure of resistive-dispersive intermediate shocks has been studied by Hau and Sonnerup (1990). A number of simulations of kinetic structure of intermediate shocks have been also carried out using hybrid codes [*Krauss-Varban et al.*, 1994; *Karimabadi et al.*, 1995]. It was shown that there exist some discrepancy between predictions of resistive MHD and kinetic results.

Now, it seems that the existence of intermediate shock has been accepted, even though criticisms persist [*Markovskii & Somov*, 1996]. Steinolfson and Hundhausen (1990) identified intermediate shocks from the numerical computation of the two-dimensional MHD equations. Even an observation of an intermediate shock has been reported by Chao *et al.* (1993). However, rigorous analytical proof of the results was still considered being unsettled. The reason is that the development of such proof involves the analytical study on the MHD Riemann problem. In particular, some questions– how intermediate shocks are generated, under what circumstances they can exist–remain unsolved.

In this paper, we try to answer these questions. Our study can be regarded as an analytical counterpart of the findings of Wu and is similar to Hada's work, but it is unique in the sense that we consider the resistive MHD equations in the context of Riemann problem and identify the nonlinear interaction of slow and fast modes. Detailed mathematical aspects can be found in a reference [*Myong & Roe,* 1997].

Analytical results

The MHD system yields seven types of wave motion whose speeds satisfy $c_s \le c_a \le c_f$, where Alfvén wave speed $c_a = \sqrt{\tau}|B_x|$ and $c_{f,s}$ are the fast and slow wave speeds, given by

$$2c_{f,s}^2/\tau = \gamma p + \mathbf{B}\cdot\mathbf{B} \pm \left[\left(\gamma p + \mathbf{B}\cdot\mathbf{B}\right)^2 - 4\gamma p B_x^2\right]^{1/2}, \quad (1)$$

where $\tau, p, \mathbf{B}, \gamma$ represent specific volume, pressure, magnetic fields, and the specific heats ratio. When acoustic wave is defined as $a^2 = \tau\gamma p$ and the magnitude of transverse magnetic field vanishes ,

$$c_s^2 = c_a^2, \quad c_f^2 = a^2 \quad \text{for} \quad a^2 > c_a^2, \quad (2)$$

$$c_f^2 = c_a^2, \quad c_s^2 = a^2 \quad \text{for} \quad a^2 < c_a^2. \quad (3)$$

The Alfvén wave interacts with one of the magnetoacoustic waves. When $c_a^2 = a^2$, it reduces to

$$c_s^2 = c_a^2 = c_f^2 = a^2. \quad (4)$$

This point is called the *umbilic point*, and represents the interaction of all three waves.

MHD shock waves

It is well known that the MHD Hugoniot and wave trajectories are all either coplanar or purely rotational. Therefore we may consider the MHD shock relations only in $(\tau, u, v, B_\perp, p)$ phase space, where $B_\perp^2 = B_y^2 + B_z^2$ and u, v represent the velocity. If τ depends only on p and $B_\perp$, it follows from (1) that $c_{f,s}$ are determined only by p and $B_\perp$ as well. Thus the following non-dimensional variables can be chosen.

$$\mathbf{U} \equiv \left(U = \frac{\gamma p}{B_x^2} = \frac{a^2}{c_a^2},\ V = \frac{B_\perp}{B_x}\right). \quad (5)$$

Then a new symmetric Rankine-Hugoniot relation can be found for one-directional shocks,

$$[U_t]\Big(\gamma(\gamma-1)[V]^3/4 + \gamma\bar{U}[V] - \bar{V}[U]\Big) - \gamma[U][V] = 0, \quad (6)$$

where $[U_t] = [U] + \gamma\bar{V}[V]$ and $[Q], \bar{Q}$ denote $(Q_R - Q_L)$ and $(Q_R + Q_L)/2$. L, R denote the downstream and upstream

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states. It can be readily extended to determine the shock jump, since it is only quadratic in U.

$$\Omega \equiv \frac{\tau}{\tau_L} = \frac{4\gamma\bar{U} + \gamma(\gamma-1)[V]^2 - 2[U]}{4\gamma\bar{U} + \gamma(\gamma-1)[V]^2 + 2[U]}, \qquad (7)$$

$$\frac{[u]}{c_{a\,L}} = -\epsilon\sqrt{\frac{(1-\Omega)[U_t]}{\gamma}},\ \frac{[v]}{c_{a\,L}} = -\epsilon[V]\sqrt{\frac{\gamma(1-\Omega)}{[U_t]}}, \qquad (8)$$

where the mass flux defined by $m = (u-s)/\tau$ becomes $\tau_L m/c_{a\,L} = -\epsilon\sqrt{[U_t]/(1-\Omega)/\gamma}$ and s represents the shock speed. For right-running waves and left-running waves, ϵ is 1 and -1, respectively.

The following parameters determine the types of shocks in planar and non-planar problems:

$$\left(\frac{c_{f,s}}{\tau}\right)_{L,R} - |m|, \quad \left(\frac{c_{f,a,s}}{\tau}\right)_{L,R} - |m|. \qquad (9)$$

They depend only on $\mathbf{U}$. The shocks can be classified by $1-4$ defined in the moving frame as

$$4 \quad c_s \quad 3 \quad c_a \quad 2 \quad c_f \quad 1.$$

Among 16 possible shocks, 12 can actually arise in MHD, but only 6 satisfy the second law of thermodynamics. They are illustrated in Figs. 1 and 2. Notice that $3 \to 4$ and $1 \to 2$ is the same as $2 \to 4$ and $1 \to 3$ in planar problem. The MHD Hugoniot is purely topological, since it can be shown that the way certain types of shocks appear is unique.

Shock admissibility and compound waves

Some of 6 entropy-satisfying shocks may not be physically relevant since the MHD system has higher degree of freedom, 2 (planar) or 3 (non-planar). Here we propose a dissipation admissibility condition in which relevant shocks are regarded as limits of travelling waves for the resistive MHD. In planar problem, singularities on the MHD dynamical system are determined by

$$\gamma(\tau_L^2 m^2/c_{aL}^2)(\Omega - 1) + [U_t] = 0, \qquad (10)$$

$$(\tau_L^2 m^2/c_{aL}^2)\big(\Omega[V] + V_L(\Omega-1)\big) - [V] = 0. \qquad (11)$$

In general, this planar vector field has four singularities [*Kennel et al.*, 1989]. Also, it can be shown by Poincaré transformation [*Perko*, 1993] that due to the presence of the umbilic point there are two singularities at infinity if $\gamma > 1$. These are nothing but hydrodynamic singularities, so that they all are nodes. It follows from Poincaré index theorem on planar vector field ($2N - 2S + 2 = 2$, $N + S = 4$) that there exist two nodes and two saddles. In this configuration, it is impossible to connect two saddles, meaning that $2 \to 3$ cannot have viscous profiles. Consequently the vector field is structurally stable by Peixoto's theorem. Thus the results may be valid even in situations deviating from the assumption of current analysis, equal dissipation coefficients. A global phase portrait is given in Fig. 3. In addition to $1 \to 2$ and $3 \to 4$, three intermediate shocks ($1 \to 3, 2 \to 4, 1 \to 4$) have viscous profiles.

In non-planar problem, global analysis is not available. The dynamical system becomes structurally unstable since a local analysis based on (9) shows the existence of saddle-saddle connections. Thus, as reported by Wu (1995), all four intermediate shocks including $2 \to 3$ have viscous profiles.

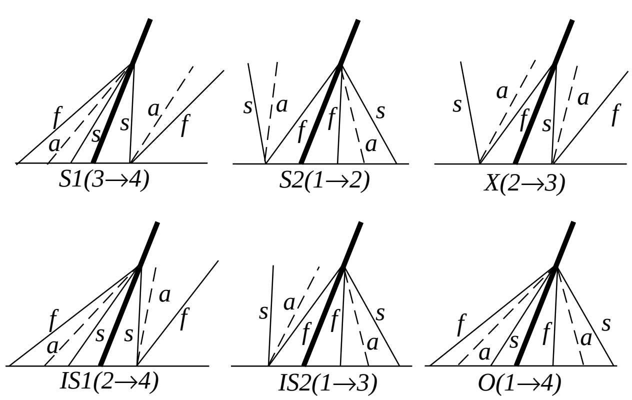

Figure 1. The relationship between the speed of entropy-satisfying shocks and characteristics in (x,t) space. $S_{1,2}$ represent slow and fast shocks, while $IS_{1,2}, X, O$ represent intermediate shocks.

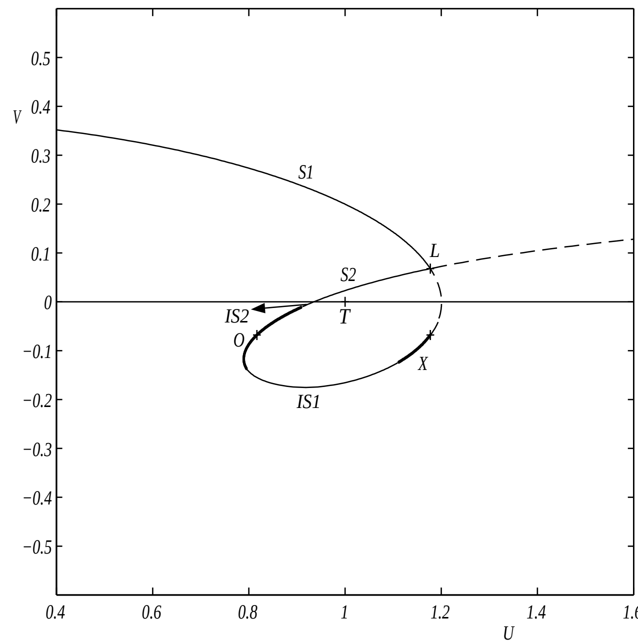

Figure 2. MHD Hugoniot loci and shock types ($\gamma = 5/3$). Entropy-violating shocks ($[U] > 0$) are denoted by dotted lines. T represents the umbilic point.

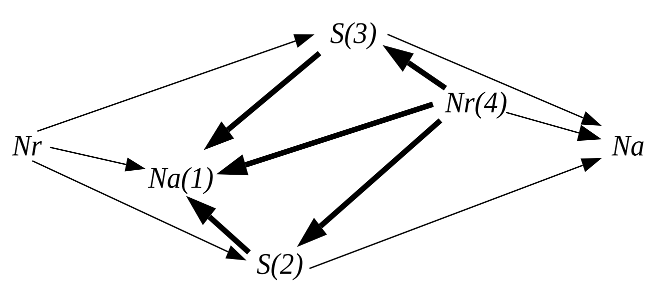

Figure 3. The MHD planar phase portrait in (U,V) space. The connections from infinity are indicated by thin solid lines. $N_r(N_a)$ and S represent repelling (attracting) nodes and saddles.

Once intermediate shocks exist, there are chances that parameters (9) vanish. Such cases can be identified if we know the behavior of rarefaction waves in $\mathbf{U}$ space. It is known that they are defined by invariants

$$J_{f,s} = U(q-1)^{-\alpha} + \alpha \int q^{-2}(q-1)^{-(\alpha+1)}\, dq, \qquad (12)$$

where $q(U,V) = c_{f,s}^2/a^2$ $(0 < q_s \leq 1, 1 \leq q_f < \infty)$ and $\alpha = \gamma/(2-\gamma)$. When $\gamma = 2$, $J_{f,s} = q_{f,s}$ The general appearance is similar to the parabola. It can be shown that the jumps, likewise the shock jumps, are determined only in $\mathbf{U}$ space. Using this information, it can be shown that parameters (9) vanish at three points in Fig. 4, B_L, D_L satisfying $c_s(\mathbf{U}_L) = \tau_L|m(\mathbf{U}_L, \mathbf{U})|$, and C_L satisfying $c_f(\mathbf{U}) = \tau|m(\mathbf{U}_L, \mathbf{U})|$. The point D_L defines a slow rarefaction wave R_1 followed by an intermediate shock IS_1, which is called a slow compound wave C_1. Similarly, the point C_L defines an intermediate shock IS_2 followed by a fast rarefaction wave R_2, which is called a fast compound wave C_2. Those compound waves are responsible for the inadmissibility of $2 \rightarrow 3$ intermediate shocks in planar problem. However, the case for B_L is different since B_L always lies between two admissible shocks (IS_2, O). In Figs. 4 and 5, physically admissible waves in planar problem including compound waves are illustrated.

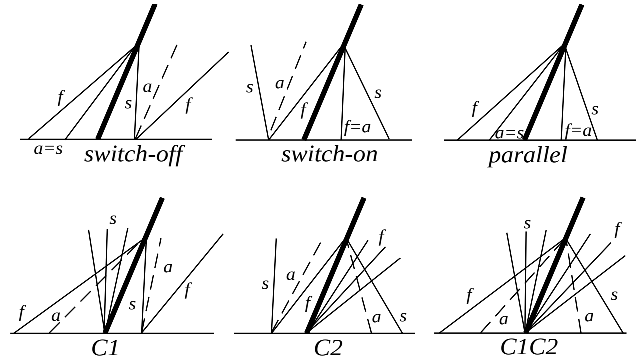

Figure 5. The configuration of compound waves and special shocks in (x,t) space.

Riemann problem

The Riemann problem concerns the evolution of an arbitrary initial discontinuity. Using wave curves, we can construct the analytical solution. For a given downstream state, by moving an intermediate state along R_1 and S_1, we can determine a fast wave that connects with a particular upstream state. An example is given in Fig. 6. Notice that the change of magnetic field always involves intermediate shocks. Up to now, only half of the Riemann problem is considered. In the full Riemann problem, the solution consists of two one-directional solutions, for instance, $C_1 R_2$ for left-running waves and $S_1 R_2$ for right-running waves. If the contact discontinuity (C) is present, then the solution is $R_2\, C_1\, C\, S_1\, R_2$. In this process, a wave combination will determine uniquely the velocity differences since all jumps depend wholly on $\mathbf{U}$. In other words, a particular set of three points in $\mathbf{U}$ space is just enough to determine the Riemann solution.

In non-planar problem, the Riemann problem becomes ill-posed. This can be easily seen by noticing that two different solutions, one as it stands $(C_1 R_2)$ and another $(R_1 R_d R_2)$ obtained by inserting a rotational discontinuity (R_d), are possible for an initial discontinuity. They involve different internal structures. Therefore, as Wu argued, it is necessary to specify precisely the local structure of intermediate shocks. Finally, it should be mentioned that the second solution may not be possible in some cases. Such cases involve switch-on, switch-off, and parallel shocks.

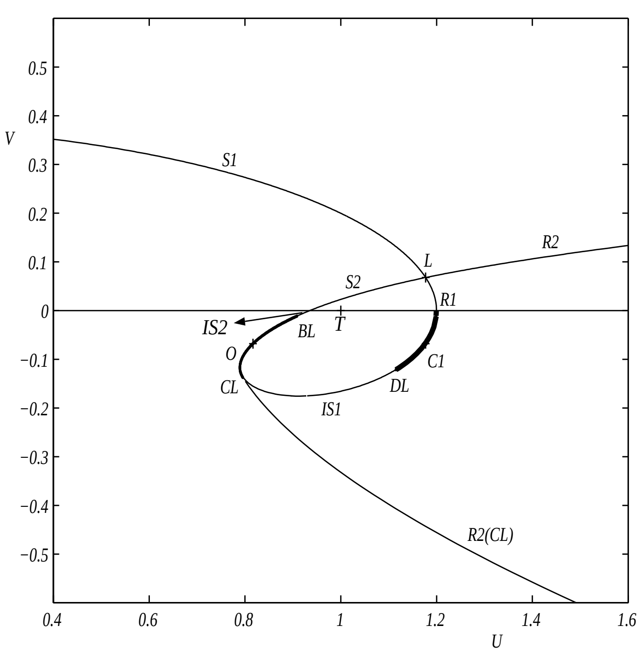

Figure 4. MHD wave curves.

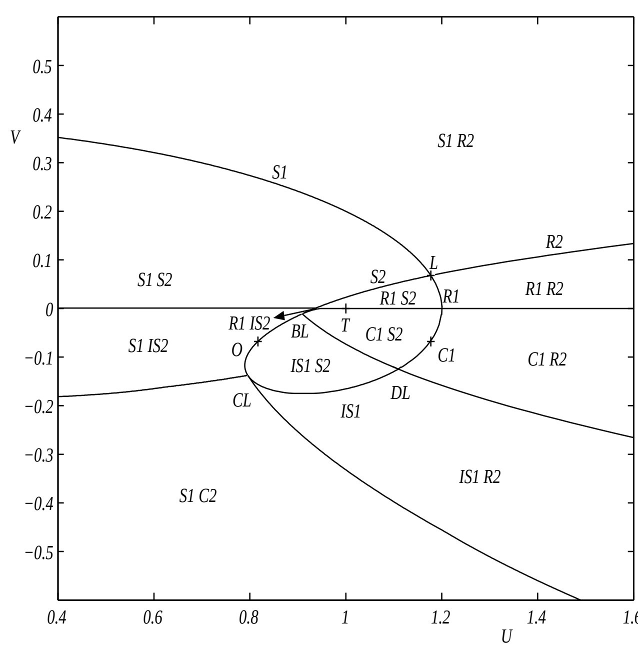

Figure 6. A Riemann solution.

Summary and discussion

We presented an analytical theory on MHD intermediate shocks in the context of Riemann problem. On a phase space, the shock properties have been described in more clear manner. It is expected that more information can be drawn once observational data are interpreted using these variables. The present theory can also describe all pictures of the evolution of shock waves arising in the resistive MHD including intermediate shocks for an initial discontinuity separating two plasmas. According to this theory, intermediate shocks-some of which belong to slow and fast compound waves-involve in the magnetic reconnection process, interplanetary shock interaction, and cometary shocks, rather than rotational discontinuities.

For the physical meaning of ill-posedness of the nonplanar Riemann problem, it seems that a convincing explanation is not yet available. However, the fact that intermediate shocks experience the relatively high entropy dissipation rate can lead to an explanation: the nonlinear evolution can be described only by planar shocks at the very large time, but intermediate shocks, time-dependent in general, are need to explain intermediate-time behavior.

On the other hand, once the effect of anisotropic pressure is included, there exist reverse-MHD regions in which slow wave speed exceeds intermediate wave speed. Some works on this problem have been reported [*Hau & Sonnerup*, 1993]. The present analysis is valid for the singularity in which two waves coincide. It is not clear to what extent the present results will remain unchanged. Clearly, further investigations into these problems including kinetic descriptions will reveal more accurate descriptions of intermediate shocks.

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