On Godunov-Type Schemes for Magnetohydrodynamics

1. A Model System

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In the light of recent analytical results on the MHD Riemann problem, Godunov-type numerical schemes for magnetohydrodynamics (MHD) are revisited. As the first step, a model system that exactly preserves the MHD hyperbolic singularities is considered. For this model, analytical results on shock waves are summarized and critical problems occurring in developing shock-capturing methods are identified. Using the results, we propose a new way to define fluxes on cell interfaces. It consists of two solvers, one on the well-posed Riemann problem and another on the evolution of Alfvén waves. Numerical experiments show that the new scheme is more efficient in calculating large-time solutions.

Key Words: Godunov-type schemes; conservation laws, magnetohydrodynamics.

1. INTRODUCTION

As numerical simulations come to play a vital role in studying the motion of ionized gases in hypersonic flows [4, 33], space propulsion [42], and space physics [15], great emphasis has been placed on the development of numerical schemes for the system of magnetohydrodynamic equations. Out of the various possibilities, Godunov-type schemes—encouraged by successful applications to the Euler equations—are considered to be highly effective in resolving discontinuities such as shock waves for high speed flow problems. These schemes were pioneered by Godunov [22] who used the local characteristic structure obtained from