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Analytical results on MHD intermediate shocks

R. S. Myong

NASA Goddard Space Flight Center, Mail Stop 930, Greenbelt, MD

Abstract. The admissibility of MHD intermediate shocks has been a matter of much debate over the years. Though the legitimacy of such shocks was shown in a recent series of investigations by Wu, rigorous analytical proof of the results was considered being unsettled. In this paper, as a step towards developing such proof, we present a theory of MHD shocks. On the basis of global analysis, we develop a shock admissibility condition. Using this theory, we explain how intermediate shocks and compound waves are generated.

Introduction

The nature of shock waves is of importance to the understanding of physical phenomena arising in various mediums. Since shock waves are omnipresent in space plasmas, for example, magnetic field reconnection [*Lin & Lee*, 1994], they have been studied with great intensity. However, the mathematical analysis involves awkward algebra, so that their exact properties are not well understood.

In the past, the evolutionary theory was used to select physically relevant shocks [*Akhiezer et al.*, 1959]. According to this theory, intermediate shocks, which can be defined as shocks that lead to a transition from super-Alfvénic to sub-Alfvénic flow, are considered inadmissible. In the framework of this theory, the rotation of transverse magnetic field is achieved only by the rotational discontinuity.

Contrary to this theory, it was shown by the pioneering numerical experiments of Wu (1987) that some intermediate shocks can exist, whereas the rotational discontinuity cannot exist. A similar conclusion was drawn in the study of evolutionary conditions of intermediate shocks by Hada (1994). Wu (1995) also found that the evolution and structure of intermediate shocks are related, so that the global solutions can be affected by the local structure. Furthermore, the structure of resistive-dispersive intermediate shocks has been studied by Hau and Sonnerup (1990). A number of simulations of kinetic structure of intermediate shocks have been also carried out using hybrid codes [*Krauss-Varban et al.*, 1994; *Karimabadi et al.*, 1995]. It was shown that there exist some discrepancy between predictions of resistive MHD and kinetic results.

Now, it seems that the existence of intermediate shock has been accepted, even though criticisms persist [*Markovskii & Somov*, 1996]. Steinolfson and Hundhausen (1990) identified intermediate shocks from the numerical computation of the two-dimensional MHD equations. Even an observation of an intermediate shock has been reported by Chao *et al.* (1993). However, rigorous analytical proof of the results was still considered being unsettled. The reason is that the development of such proof involves the analytical study on the MHD Riemann problem. In particular, some questions– how intermediate shocks are generated, under what circumstances they can exist–remain unsolved.

In this paper, we try to answer these questions. Our study can be regarded as an analytical counterpart of the findings of Wu and is similar to Hada's work, but it is unique in the sense that we consider the resistive MHD equations in the context of Riemann problem and identify the nonlinear interaction of slow and fast modes. Detailed mathematical aspects can be found in a reference [*Myong & Roe,* 1997].

Analytical results

The MHD system yields seven types of wave motion whose speeds satisfy $c_s \leq c_a \leq c_f$, where Alfvén wave speed $c_a = \sqrt{\tau}|B_x|$ and $c_{f,s}$ are the fast and slow wave speeds, given by

$$2c_{f,s}^2/\tau = \gamma p + \mathbf{B}\cdot\mathbf{B} \pm \left[\left(\gamma p + \mathbf{B}\cdot\mathbf{B}\right)^2 - 4\gamma p B_x^2\right]^{1/2}, \quad (1)$$

where $\tau, p, \mathbf{B}, \gamma$ represent specific volume, pressure, magnetic fields, and the specific heats ratio. When acoustic wave is defined as $a^2 = \tau\gamma p$ and the magnitude of transverse magnetic field vanishes ,

$$c_s^2 = c_a^2, \quad c_f^2 = a^2 \quad \text{for} \quad a^2 > c_a^2, \quad (2)$$

$$c_f^2 = c_a^2, \quad c_s^2 = a^2 \quad \text{for} \quad a^2 < c_a^2. \quad (3)$$

The Alfvén wave interacts with one of the magnetoacoustic waves. When $c_a^2 = a^2$, it reduces to

$$c_s^2 = c_a^2 = c_f^2 = a^2. \quad (4)$$

This point is called the *umbilic point*, and represents the interaction of all three waves.

MHD shock waves

It is well known that the MHD Hugoniot and wave trajectories are all either coplanar or purely rotational. Therefore we may consider the MHD shock relations only in $(\tau, u, v, B_\perp, p)$ phase space, where $B_\perp^2 = B_y^2 + B_z^2$ and u, v represent the velocity. If τ depends only on p and $B_\perp$, it follows from (1) that $c_{f,s}$ are determined only by p and $B_\perp$ as well. Thus the following non-dimensional variables can be chosen.

$$\mathbf{U} \equiv \left(U = \frac{\gamma p}{B_x^2} = \frac{a^2}{c_a^2},\ V = \frac{B_\perp}{B_x}\right). \quad (5)$$

Then a new symmetric Rankine-Hugoniot relation can be found for one-directional shocks,

$$[U_t]\Big(\gamma(\gamma-1)[V]^3/4 + \gamma\bar{U}[V] - \bar{V}[U]\Big) - \gamma[U][V] = 0, \quad (6)$$

where $[U_t] = [U] + \gamma\bar{V}[V]$ and $[Q], \bar{Q}$ denote $(Q_R - Q_L)$ and $(Q_R + Q_L)/2$. L, R denote the downstream and upstream

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