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ORIGINAL ARTICLE

R. S. Myong

Coupled nonlinear constitutive models for rarefied and microscale gas flows: subtle interplay of kinematics and dissipation effects

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Abstract The constitutive relations of gases in a thermal nonequilibrium (rarefied and microscale) can be derived by applying the moment method to the Boltzmann equation. In this work, a model constitutive relation determined on the basis of the moment method is developed and applied to some challenging problems in which classical hydrodynamic theories including the Navier–Stokes–Fourier theory are shown to predict qualitatively wrong results. Analysis of coupled nonlinear constitutive models enables the fundamentals of gas flows in thermal nonequilibrium to be identified: namely, nonlinear, asymmetric, and coupled relations between stresses and the shear rate; and effect of the bulk viscosity. In addition, the new theory explains the central minimum of the temperature profile in a force-driven Poiseuille gas flow, which is a well-known problem that renders the classical hydrodynamic theory a global failure.

Keywords Continuum mechanics · Constitutive relation · Microscale and nanoscale gases · Force-driven Poiseuille flow

PACS 47.10.ab Conservation laws and constitutive relations · 47.45.Ab Kinetic theory of gases · 47.61.Cb Non-continuum effects

1 Introduction

A constitutive equation represents the relation between the microscopic and macroscopic physics of a specific substance of interest. As the only ingredient that appears in the continuum mechanics intended to bridge two distinctive worlds (macroscopic and microscopic), it plays a critical role in describing the structure of substance. Depending on the type of materials, the constitutive equation can generally take various forms: for example, Hooke's law for pure solid materials which has been studied in previous works [1]. On the other hand, less attention has been directed to the development of the corresponding equations for gases; until recently, virtually all the practical flow and heat transfer problems were dealt in terms of the classical model known as the Navier–Stokes–Fourier model. However, research interests have shifted from the classical problem to the more challenging problem that arises from the micromechanics and nanomechanics of materials related to recent developments in nanotechnology and nanoscience: accordingly, there is a growing need to develop new constitutive equations for rarefied and microscale gas flows [2].

That goal is pursued here by applying the so-called moment method [3,4] to the Boltzmann equation, which is derived from the kinetic viewpoint of gas motion. With the Boltzmann equation as the starting point, a new model of the constitutive relation can be developed and applied to challenging problems, particularly those where the classical hydrodynamic theories including the Navier–Stokes–Fourier theory are shown to

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R. S. Myong
Department of Mechanical and Aerospace Engineering and Research Center for Aircraft Parts Technology,
Gyeongsang National University, Jinju, Kyeongnam 660-701, South Korea
E-mail: myong@gnu.ac.kr