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A second-order positivity-preserving finite volume upwind scheme for air-mixed droplet flow in atmospheric icing

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ABSTRACT

A second-order positivity-preserving finite volume upwind scheme based on the approximate Riemann solver is developed for computing the Eulerian two-phase flow composed of air and small water droplets in atmospheric icing. In order to circumvent a numerical problem due to the non-strictly hyperbolic nature of the original Eulerian droplet equations, a simple technique based on splitting of the original system into the well-posed hyperbolic part and the source term is proposed. The positivity-preserving Harten–Lax–van Leer-Contact approximate Riemann solver is then applied to the well-posed hyperbolic part of the Eulerian droplet equations. It is demonstrated that the new scheme satisfies the positivity condition for the liquid water contents. The numerical results of one and two-dimensional test problems are also presented as the verification and validation of the new scheme. Lastly, the exact analytical Riemann solutions of the well-posed hyperbolic part of the droplet equations in wet and dry regions are given for the verification study.

1. Introduction

Two-phase flows are very common in many scientific and engineering applications ranging from fluid dynamics, chemical engineering, combustion and many more. Accurate mathematical and computational modeling of these two-phase flow phenomena is currently a very active field of research. For example, Toumi and Kumbaro [1] developed an approximate linearized Riemann solver for the numerical simulation of an isentropic two-component two-phase flow. Saurel and Abgrall [2] also proposed a new isentropic two-phase model and an associated Godunov method for compressible multi-fluid and multi-phase flows. It was later recognized by Tian et al. [3] that the compressible two-phase model equations cannot always be written in conservative form, though they may be hyperbolic. In order to resolve this problem, they developed a path-conservative method for a five-equation model of two-phase flow with an approximate Harten–Lax–van Leer-Contact (HLLC)-type Riemann solver. In these works, two-phase flows are treated in a strongly coupled manner so that the equations of each phase are fully solved and thus both phases (gas and liquid) affect each other strongly, even though no viscous effects are assumed to be present in the physical system.

There is a very important two-phase flow in which a drastic physical simplification is possible. Such a case is found in the air-mixed droplet flow field that describes the gas–liquid two-phase diluted flows around aircraft flying inside a cloud composed of compressible air and small super-cooled droplets of liquid water in the atmosphere. This is called *atmospheric in-flight icing* in the field and remains as a critical technological issue in the safety of aircraft [4–6]. A similar problem can be found in wind turbine blades operating in cold climate. It is called simply *atmospheric icing* in the field [7]. The two-phase flow in those situations can be simulated using a weakly coupled (or one-way coupling) algorithm since the effects of a droplet on the air flow can be ignored. In general, the mass loading ratio of the bulk density of the droplets over the bulk density of air is on the order of 10^{-3} under icing conditions. This observation can therefore justify a weakly coupled algorithm in which separate calculations are made of the atmosphere cloud condition mixed with the air and a super-cooled water droplet. Under this condition, the Eulerian droplet equations of two-phase flow can be written as [8,9]

$$\begin{aligned} &\rho_t + \nabla\cdot(\rho\mathbf{u}) = 0, \\ &(\rho\mathbf{u})_t + \nabla\cdot(\rho\mathbf{u}\mathbf{u}) = \mathbf{S}(\rho,\rho_g,T_g,\mathbf{u},\mathbf{u}_g,\mathbf{g}). \end{aligned} \tag{1.1}$$

In these *convection-type* equations with no diffusion terms, the ρ and $\mathbf{u}$ are the droplet density in terms of liquid water content (LWC) and the velocity of the droplet, respectively. The ρ_g, T_g, $\mathbf{u}_g$ and $\mathbf{g}$ are the air density, the air temperature, the air velocity, and the acceleration vector due to gravity, respectively. The vector $\mathbf{S}$ represents the source effects which include the aerodynamic drag term, gravity term, and buoyancy term of the droplet. The effects

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