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A triangular discontinuous Galerkin method for non-Newtonian implicit constitutive models of rarefied and microscale gases

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ABSTRACT

The discontinuous Galerkin (DG) method has been popular as a numerical technique for solving the conservation laws of gas dynamics. In the present study, we develop an explicit modal DG scheme for multi-dimensional conservation laws on unstructured triangular meshes in conjunction with non-Newtonian implicit nonlinear coupled constitutive relations (NCCR). Special attention is given to how to treat the complex non-Newtonian type constitutive relations arising from the high degree of thermal nonequilibrium in multi-dimensional gas flows within the Galerkin framework. The Langmuir velocity slip and temperature jump conditions are also implemented into the two-dimensional DG scheme for high Knudsen number flows. As a canonical scalar case, Newtonian and non-Newtonian convection–diffusion Burgers equations are studied to develop the basic building blocks for the scheme. In order to verify and validate the scheme, we applied the scheme to a stiff problem of the shock wave structure for all Mach numbers and to the two-dimensional hypersonic rarefied and low-speed microscale gas flows past a circular cylinder. The computational results show that the NCCR model yields the solutions in better agreement with the direct simulation Monte Carlo (DSMC) data than the Newtonian linear Navier–Stokes–Fourier (NSF) results in all cases of the problem studied.

1. Introduction

In spite of considerable efforts over the past decades, an accurate numerical simulation of nonequilibrium rarefied and microscale gases remains very challenging in the field of computational fluid dynamics (CFD). Nonequilibrium gas flows of technological interest may be found in three distinctive problems [1–4]: 1) the hypersonic rarefied regime of trans-atmospheric vehicles; 2) the microscale low-speed (creeping) regime of micro- and nano-devices; and 3) the high-speed free-molecular regime of mechanical devices operating near the vacuum condition. The high Mach and Knudsen numbers are the sources of thermal nonequilibrium in these cases (the high Mach, the high Knudsen, or both). From past experiences, it was accepted that conventional models based on classical physics, such as the NSF equations, have serious limitations in capturing the correct flow physics of high thermal nonequilibrium. In the NSF equations, non-conserved variables associated with thermal nonequilibrium, the shear stress tensor and heat flux vector, are described in conjunction with the *linear*

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