Physics of polyatomic gases in non-equilibrium based on the second-order Boltzmann-type kinetic theory

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Outline

Part I

2^{nd}-order Boltzmann-type kinetic theory: Boltzmann-based gas dynamics (BGD):

Part II

2^{nd}-order constitutive laws for polyatomic gases

Physics of polyatomic gases in non-equilibrium via

- Topological representation of constitutive laws
- Multi-dimensional flow fields obtained by CFD based on discontinuous Galerkin method

2nd-order Boltzmann-type kinetic theory

PoF 1999, JCP 2001, JCP 2004: Eu's generalized hydrodynamics

PoF 2014, PoF 2016: Balanced closure & validation via MD

PoF 2018: Polyatomic gases (shock-vortex interaction)

Conceptual revision

New closure theory

Physical insight

More validation

Discontinuous Galerkin

Basic information can be found in the Youtube of an Indian GIAN (Global Initiative Academic Networks) Lecture (2017; IIT Kanpur; 15 Lectures)

Rarefied & Microscale Gases and Viscoelastic Fluids: A Unified Framework

https://www.youtube.com/ and search "Rarefied & Microscale Gases"

Other independent NCCR works: Multi-species extension by Ahn & Kim (SNU, Korea, JCP09)

Implicit-FVM implementation by Jiang & Zhao (Zhejiang Univ., China, 2017)

Cf. **NCCR**: Nonlinear Coupled Constitutive Relation

Closure-last moment method

$$\rho \mathbf{u} = \iiint m\mathbf{v} f(t, \mathbf{r}, \mathbf{v}) dv_x dv_y dv_z$$

Molecular			Continuum	Molecular				Continuum
Pure (or Semi-) Simulation				**PDE-based Approach**				
MD	DSMC	Gas-Kinetic Scheme	LBM	Liouville Equation	Boltzmann and Simplified Boltzmann	Method of Moments	Chapman-Enskog and Burnett	NSF
Microscopic	Mesoscopic		Macroscopic	Microscopic		Mesoscopic		Macroscopic

Breakdown of moment method: 1) when the statistical average is meaningless due to too few particles; 2) when thermodynamics is not definable.

Closure-first approach: Grad's 13 moment method (1949) based on **polynomial** expansion

Levermore method (1996) based on **Gaussian** (exponential) expansion

Regularized-13 moment method (2003)

Closure-last approach: Eu's generalized hydrodynamics (1980) based on

Eu's closure & **equation of transfer**

NCCR (2014) based on **balanced closure** & **equation of transfer**

Boltzmann transport equation (BTE)

- A first-order **partial differential** equation with an **integral** collision term

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)f(t,\mathbf{r},\mathbf{v})=\frac{1}{Kn}C[f,f_2]$$

Movement   Collision (or Interaction)

Kinematic   Dissipation

$$C[f,f_2]\sim\int|\mathbf{v}-\mathbf{v}_2|(f^*f_2^*-ff_2)d\mathbf{v}_2$$

$$= \text{Gain (scattered into) - Loss (scattered out)} = \left(\frac{\delta f}{\delta t}\right)^+-\left(\frac{\delta f}{\delta t}\right)^-$$

- **Maxwell's equation of transfer** for molecular expression $h^{(n)}$

$$\frac{\partial}{\partial t}\left\langle h^{(n)}f\right\rangle+\nabla\cdot\left(\mathbf{u}\left\langle h^{(n)}f\right\rangle+\left\langle \mathbf{c}h^{(n)}f\right\rangle\right)-\left\langle f\frac{d}{dt}h^{(n)}\right\rangle-\left\langle f\mathbf{c}\cdot\nabla h^{(n)}\right\rangle=\left\langle h^{(n)}C[f,f_2]\right\rangle$$

Complexity out of simplicity: conservation laws

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right) f(t,\mathbf{r},\mathbf{v}) = C[f, f_2]$$

$\rho\mathbf{u} = \langle m\mathbf{v}f(t,\mathbf{r},\mathbf{v})\rangle$

where $\langle\cdots\rangle = \iiint \cdots dv_x dv_y dv_z$

Differentiating the statistical definition $\rho\mathbf{u} \equiv \langle m\mathbf{v}f(t,\mathbf{r},\mathbf{v})\rangle$ *with time* and *then combining* with BTE ($t,\mathbf{r},\mathbf{v}$ are independent and $\mathbf{v} = \mathbf{u}+\mathbf{c}$)

$$\frac{\partial}{\partial t}\langle m\mathbf{v}f\rangle = \left\langle m\mathbf{v}\frac{\partial f}{\partial t}\right\rangle = -\langle m(\mathbf{v}\cdot\nabla f)\mathbf{v}\rangle + \langle m\mathbf{v}C[f, f_2]\rangle$$

$[\mathbf{A}]^{(2)}$: Traceless symmetric part of tensor $\mathbf{A}$

Here $-\langle m(\mathbf{v}\cdot\nabla f)\mathbf{v}\rangle = -\nabla\cdot\langle m\mathbf{v}\mathbf{v}f\rangle = -\nabla\cdot\{\rho\mathbf{u}\mathbf{u} + \langle m\mathbf{c}\mathbf{c}f\rangle\}$

After the decomposition of the stress into pressure and viscous shear stress $\mathbf{\Pi}$

$\mathbf{P} \equiv \langle m\mathbf{c}\mathbf{c}f\rangle = p\mathbf{I} + \mathbf{\Pi}$ where $p \equiv \langle m\mathrm{Tr}(\mathbf{c}\mathbf{c})f/3\rangle$, $\mathbf{\Pi} \equiv \langle m[\mathbf{c}\mathbf{c}]^{(2)} f\rangle$,

and using the collisional invariance of the momentum, $\langle m\mathbf{v}C[f, f_2]\rangle = 0$,

we have

$$\frac{\partial(\rho\mathbf{u})}{\partial t} + \nabla\cdot(\rho\mathbf{u}\mathbf{u} + p\mathbf{I} + \mathbf{\Pi}) = 0.$$

Exact consequence of the original BTE!

Further complexity: constitutive equations

For arbitrary molecular expressions of general moment
(**viscous shear stress** $\boldsymbol{\Pi}$ and **heat flux** $\mathbf{Q}$)

$$\rho \frac{D}{Dt}\begin{bmatrix} \boldsymbol{\Pi}\left(\equiv \left\langle m[\mathbf{cc}]^{(2)} f \right\rangle\right)/\rho \\ \mathbf{Q}\left(\equiv \left\langle mc^2\mathbf{c}/2 f \right\rangle\right)/\rho \end{bmatrix} + \nabla\cdot\begin{bmatrix} \langle m\mathbf{ccc}f\rangle - \langle m\mathrm{Tr}(\mathbf{ccc})f\rangle \mathbf{I}/3 \\ \left\langle mc^2\mathbf{cc}f/2\right\rangle - C_pT(p\mathbf{I}+\boldsymbol{\Pi}) \end{bmatrix} + \begin{bmatrix} 0 \\ \langle m\mathbf{ccc}f\rangle : \nabla\mathbf{u} \end{bmatrix}$$

$$\begin{bmatrix} 2[\boldsymbol{\Pi}\cdot\nabla\mathbf{u}]^{(2)} \\ \dfrac{D\mathbf{u}}{Dt}\cdot\boldsymbol{\Pi} + \mathbf{Q}\cdot\nabla\mathbf{u} + \boldsymbol{\Pi}\cdot C_p\nabla T \end{bmatrix} + \begin{bmatrix} 2p[\nabla\mathbf{u}]^{(2)} \\ pC_p\nabla T \end{bmatrix} = \begin{bmatrix} \left\langle m[\mathbf{cc}]^{(2)} C[f,f_2]\right\rangle \\ \left\langle\left(mc^2/2 - mC_pT\right)\mathbf{c}C[f,f_2]\right\rangle \end{bmatrix}$$

No approximation so far: exact consequence of the original BTE via Maxwell's equation of transfer

Balanced closure: constitutive equations

Conceptual inconsistency of Eu's closure (1992)

$$\left\langle m\mathbf{ccc}f\right\rangle-\left\langle m\mathrm{Tr}(\mathbf{ccc})\,f\right\rangle\mathbf{I}/3=0\ \Rightarrow\ \text{Vanishing heat flux}$$

$$\left\langle m\mathbf{ccc}f\right\rangle=0\ \Rightarrow\ \text{Cannot be zero in general}$$

New balanced closure with closure-last approach (2014)

2nd-order for kinematic LH = 2nd-order for collsion RH

2nd-order closure

$$\rho\frac{D}{Dt}\begin{bmatrix}\mathbf{\Pi}\left(\equiv\left\langle m[\mathbf{cc}]^{(2)}\,f\right\rangle\right)/\rho\\ \mathbf{Q}\left(\equiv\left\langle mc^2\mathbf{c}/2\,f\right\rangle\right)/\rho\end{bmatrix}+\boxed{\nabla\cdot\begin{bmatrix}\left\langle m\mathbf{ccc}f\right\rangle-\left\langle m\mathrm{Tr}(\mathbf{ccc})\,f\right\rangle\mathbf{I}/3\\ \left\langle mc^2\mathbf{cc}f/2\right\rangle-C_pT(p\mathbf{I}+\mathbf{\Pi})\end{bmatrix}+\begin{bmatrix}0\\ \left\langle m\mathbf{ccc}f\right\rangle:\nabla\mathbf{u}\end{bmatrix}}$$

$$\begin{bmatrix}2[\mathbf{\Pi}\cdot\nabla\mathbf{u}]^{(2)}\\ \dfrac{D\mathbf{u}}{Dt}\cdot\mathbf{\Pi}+\mathbf{Q}\cdot\nabla\mathbf{u}+\mathbf{\Pi}\cdot C_p\nabla T\end{bmatrix}+\begin{bmatrix}2p[\nabla\mathbf{u}]^{(2)}\\ pC_p\nabla T\end{bmatrix}=\boxed{\begin{bmatrix}\left\langle m[\mathbf{cc}]^{(2)}C[f,f_2]\right\rangle\\ \left\langle\left(mc^2/2-mC_pT\right)\mathbf{c}C[f,f_2]\right\rangle\end{bmatrix}}$$

2nd-order closure

Conservation laws + 2nd-order constitutive relations

Hyperbolic (inviscid)

Conservation laws (exact consequence of BTE)

$$\boxed{\rho \frac{D}{Dt}\begin{bmatrix} 1/\rho \\ \mathbf{u} \\ E_t \end{bmatrix} + \nabla \cdot \begin{bmatrix} -\mathbf{u} \\ p\mathbf{I} \\ p\mathbf{u} \end{bmatrix}} + \nabla \cdot \begin{bmatrix} 0 \\ \mathbf{\Pi} \\ \mathbf{\Pi}\cdot\mathbf{u} + \mathbf{Q} \end{bmatrix} = \mathbf{0}$$

in conjunction with the 2nd-order constitutive relations (CR)

$$\rho \frac{D(\mathbf{\Pi}/\rho)}{Dt} + \underbrace{2[\mathbf{\Pi}\cdot\nabla\mathbf{u}]^{(2)}}_{\text{2nd-order coupling}} + \underbrace{2p[\nabla\mathbf{u}]^{(2)}}_{\text{Navier 1st law}} = -\frac{p}{\mu_{NS}}\mathbf{\Pi}q_{2nd}(\kappa_1),$$

$$\rho \frac{D(\mathbf{Q}/\rho)}{Dt} + \frac{D\mathbf{u}}{Dt}\cdot\mathbf{\Pi} + \underbrace{\mathbf{Q}\cdot\nabla\mathbf{u} + \mathbf{\Pi}\cdot C_p\nabla T}_{\text{2nd-order coupling}} + \underbrace{C_p p\nabla T}_{\text{Fourier 1st law}} = -\frac{pC_p}{k_{NS}}\mathbf{Q}q_{2nd}(\kappa_1),$$

Non-hyperbolic

Implicit

$$q_{2nd}(\kappa_1) \equiv \frac{\sinh\kappa_1}{\kappa_1}, \quad \kappa_1 \equiv \frac{T^{1/4}}{p}\left(\frac{\mathbf{\Pi}:\mathbf{\Pi}}{\mu_{NS}} + \frac{\mathbf{Q}\cdot\mathbf{Q}/T}{k_{NS}}\right)^{1/2}$$

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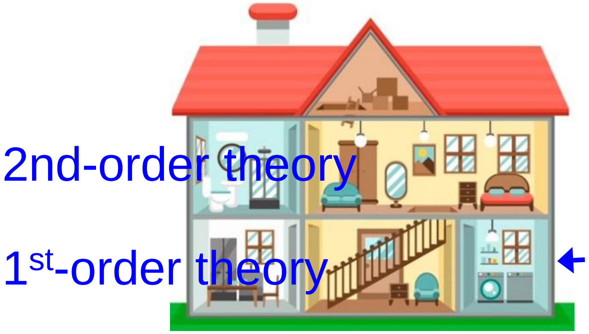
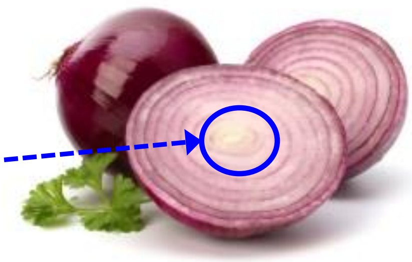
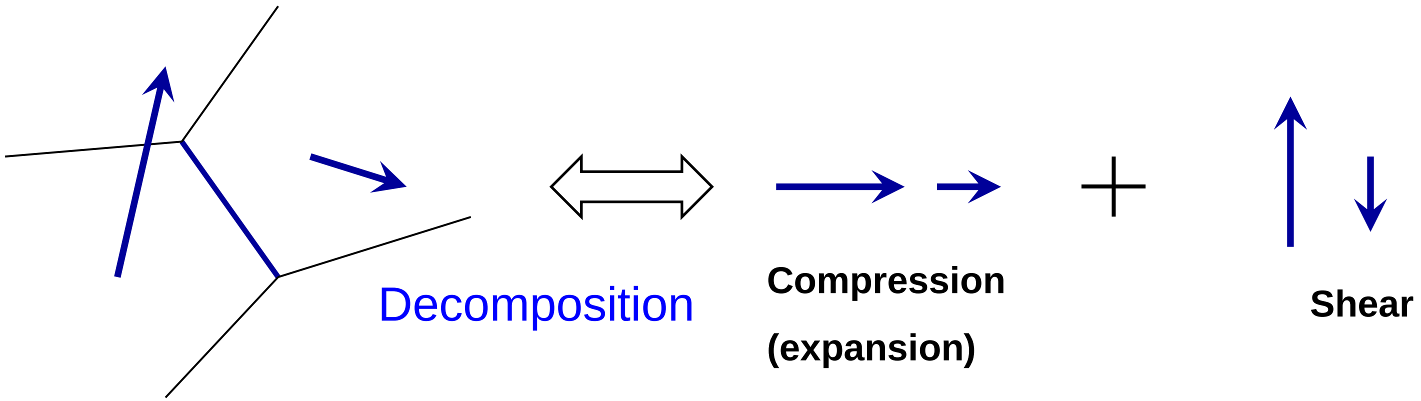
2nd-order theory in elementary flows



Navier-Fourier laws inclusive like onion!





- Critical role of $q_{2nd}(\kappa_1)$ term in compression case
- Negligible role of $q_{2nd}(\kappa_1)$ term in expansion & velocity shear (simple shear thinning – decreasing viscosity)

A model for 1-D compression & expansion

$$\rho \frac{D(\mathbf{\Pi}/\rho)}{Dt} + 2[\mathbf{\Pi}\cdot\nabla\mathbf{u}]^{(2)} + 2p[\nabla\mathbf{u}]^{(2)} = -\frac{p}{\mu_{NS}}\mathbf{\Pi}\left(\frac{\sinh\kappa_1}{\kappa_1}\right),\ \kappa_1 \equiv \frac{T^{1/4}}{p}\left(\frac{\mathbf{\Pi}:\mathbf{\Pi}}{\mu_{NS}} + \frac{\mathbf{Q}\cdot\mathbf{Q}/T}{k_{NS}}\right)^{1/2}$$

No heat flux ⇩

$$\rho \frac{D(\mathbf{\Pi}/\rho)}{Dt}(-\mu_{NS}) + [\mathbf{\Pi}\cdot(-2\mu_{NS})\nabla\mathbf{u}]^{(2)} + p(-2\mu_{NS})[\nabla\mathbf{u}]^{(2)} = p\mathbf{\Pi}\left(\frac{\sinh\kappa_1}{\kappa_1}\right),\ \kappa_1 \equiv \frac{T^{1/4}}{p}\left(\frac{\mathbf{\Pi}:\mathbf{\Pi}}{\mu_{NS}}\right)^{1/2}$$

One-dimension ⇩

$$\mathbf{\Pi}_{NS} \equiv -2\mu_{NS}[\nabla\mathbf{u}]^{(2)}$$

$$\hat{t} \equiv \frac{t}{\mu_{NS}/p},\ \hat{\Pi} \equiv \frac{\Pi}{p},\ \Pi_{NS} \equiv -\frac{4}{3}\mu_{NS}\frac{\partial u}{\partial x}$$

$$\frac{D\hat{\Pi}}{D\hat{t}} - \hat{\Pi}\hat{\Pi}_{NS} - \hat{\Pi}_{NS} = -\hat{\Pi}\,q(|\hat{\Pi}|),\ \text{where } q(|\hat{\Pi}|) \equiv \sinh|\hat{\Pi}|/|\hat{\Pi}|$$

$$t_{stress}(10^{-8}\ \text{sec}) \ll t_{flow}$$ ⇩ Or steady-state

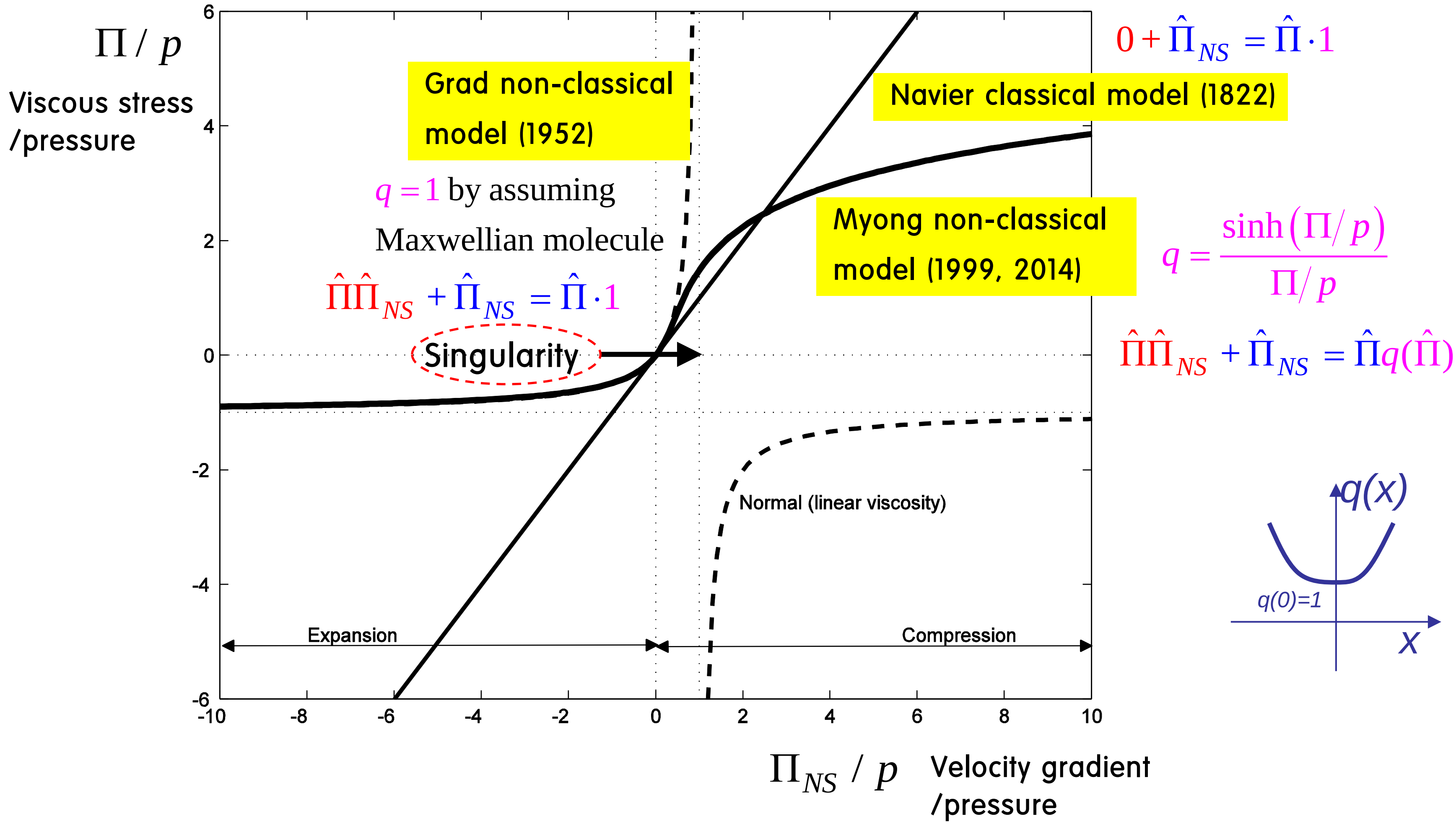
$$\boxed{\hat{\Pi}\hat{\Pi}_{NS} + \hat{\Pi}_{NS} = \hat{\Pi}\,q(|\hat{\Pi}|)}$$

2nd-order   1st-order   1st-order x 2nd-order

Kinematic motion   **Collision**

2^{nd}-order constitutive laws in compression/expansion

1-D compression and expansion



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1-D shock result: inverse shock thickness

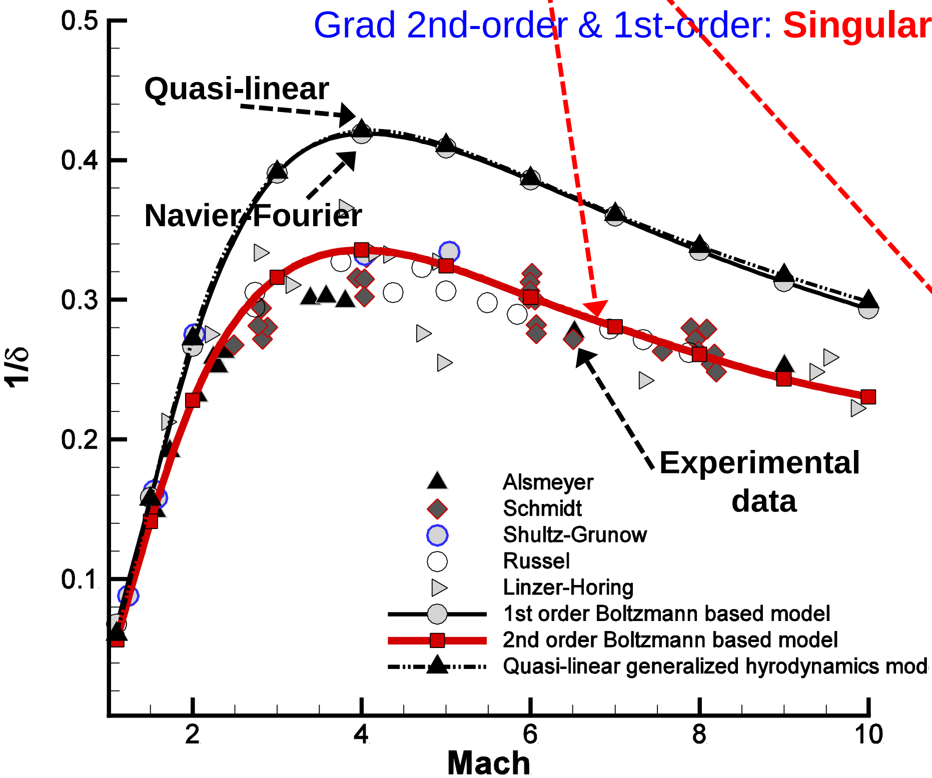
New 2nd-order & 2nd-order: **Best**

Navier 1st-order & 1st-order: Reference

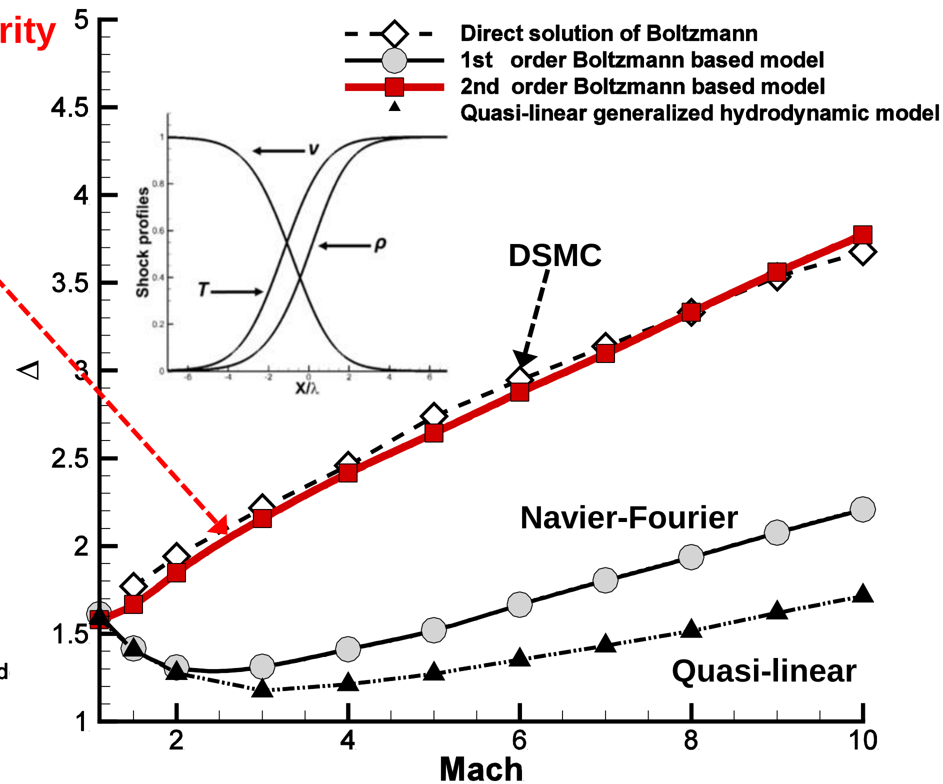
$0+\hat{\Pi}_{NS}=\hat{\Pi}q(\hat{\Pi})$ ←-- Quasi-linear 1st-order & 2nd-order: **Worse** than Navier

Grad 2nd-order & 1st-order: **Singularity**

⇨ Importance of **balanced** closure



Shock density thickness of argon gas



Shock temperature-density distance

(Karchani, *PhD Thesis*, GNU, 2017)

2^{nd}-order constitutive laws in velocity shear

$$\rho\frac{D(\boldsymbol{\Pi}/\rho)}{Dt} + 2\left[\boldsymbol{\Pi}\cdot\nabla\mathbf{u}\right]^{(2)} + 2p\left[\nabla\mathbf{u}\right]^{(2)} = -\frac{p}{\mu_{NS}}\boldsymbol{\Pi}\, q_{2nd}(\kappa_1),$$

$$\rho\frac{D(\mathbf{Q}/\rho)}{Dt} + \frac{D\mathbf{u}}{Dt}\cdot\boldsymbol{\Pi} + \mathbf{Q}\cdot\nabla\mathbf{u} + \boldsymbol{\Pi}\cdot C_p\nabla T + C_p p\nabla T = -\frac{pC_p}{k_{NS}}\mathbf{Q}\, q_{2nd}(\kappa_1)$$

1-D velocity shear → **No heat flux**

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla = 0$$

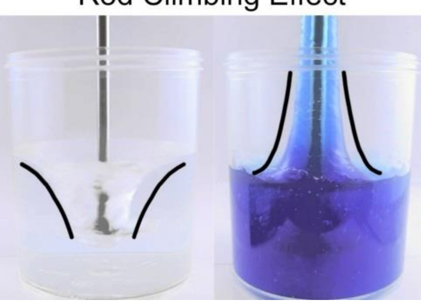
$$\frac{1}{\mu_{NS}}\begin{bmatrix} 2\Pi_{xy}\Pi_{xy_{NS}}/3 \\ -\Pi_{yy}\Pi_{xy_{NS}} \end{bmatrix} - \frac{1}{\mu_{NS}}\begin{bmatrix} 0 \\ p\Pi_{xy_{NS}} \end{bmatrix} = -\frac{p}{\mu_{NS}}\begin{bmatrix} \Pi_{yy} \\ \Pi_{xy} \end{bmatrix} q_{2nd}(\kappa_1)$$

$$\boxed{\hat{\Pi}_{xy}^2 = -\frac{3}{2}\left(1+\hat{\Pi}_{yy}\right)\hat{\Pi}_{yy}}$$

Kinematic stress constraint due to 2^{nd}-order coupling!

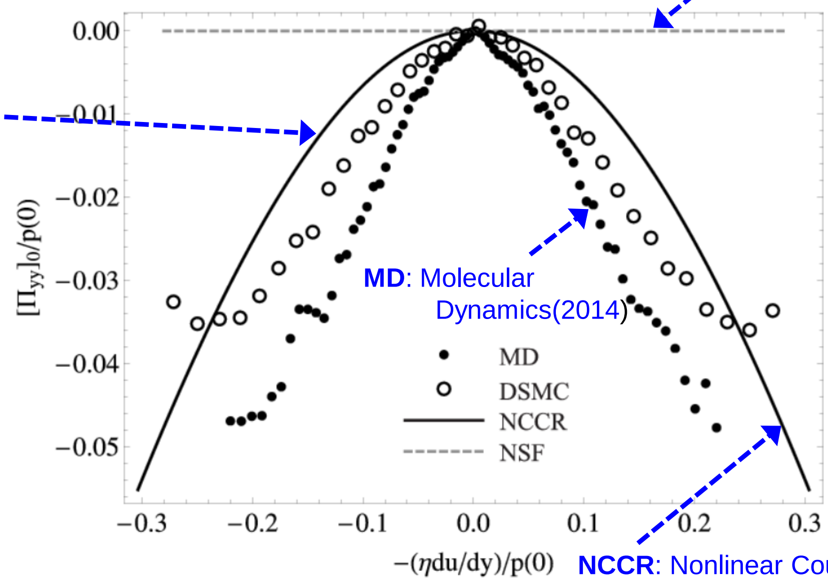
No such thing in 1^{st}-order Navier law!

Rod Climbing Effect



Newtonian **Visco-elastic**

NSF: Navier-Stokes-Fourier



NCCR: Nonlinear Coupled Constitutive Relation

Multi-dimensional CFD based on DG

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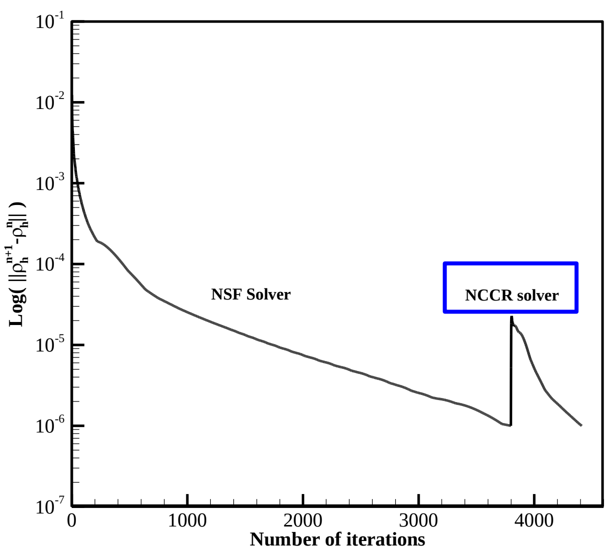
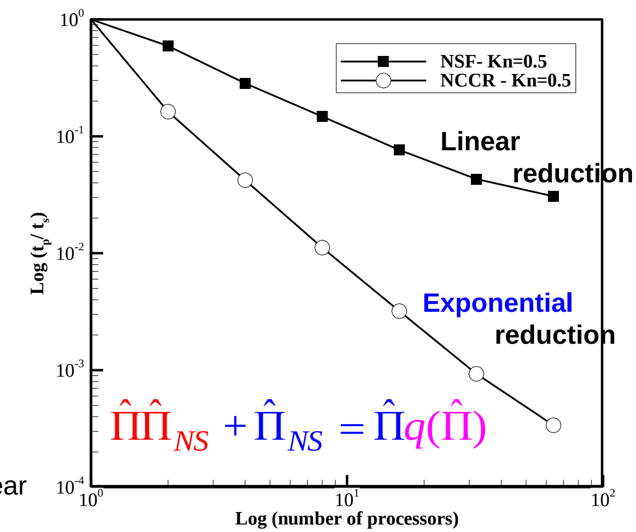


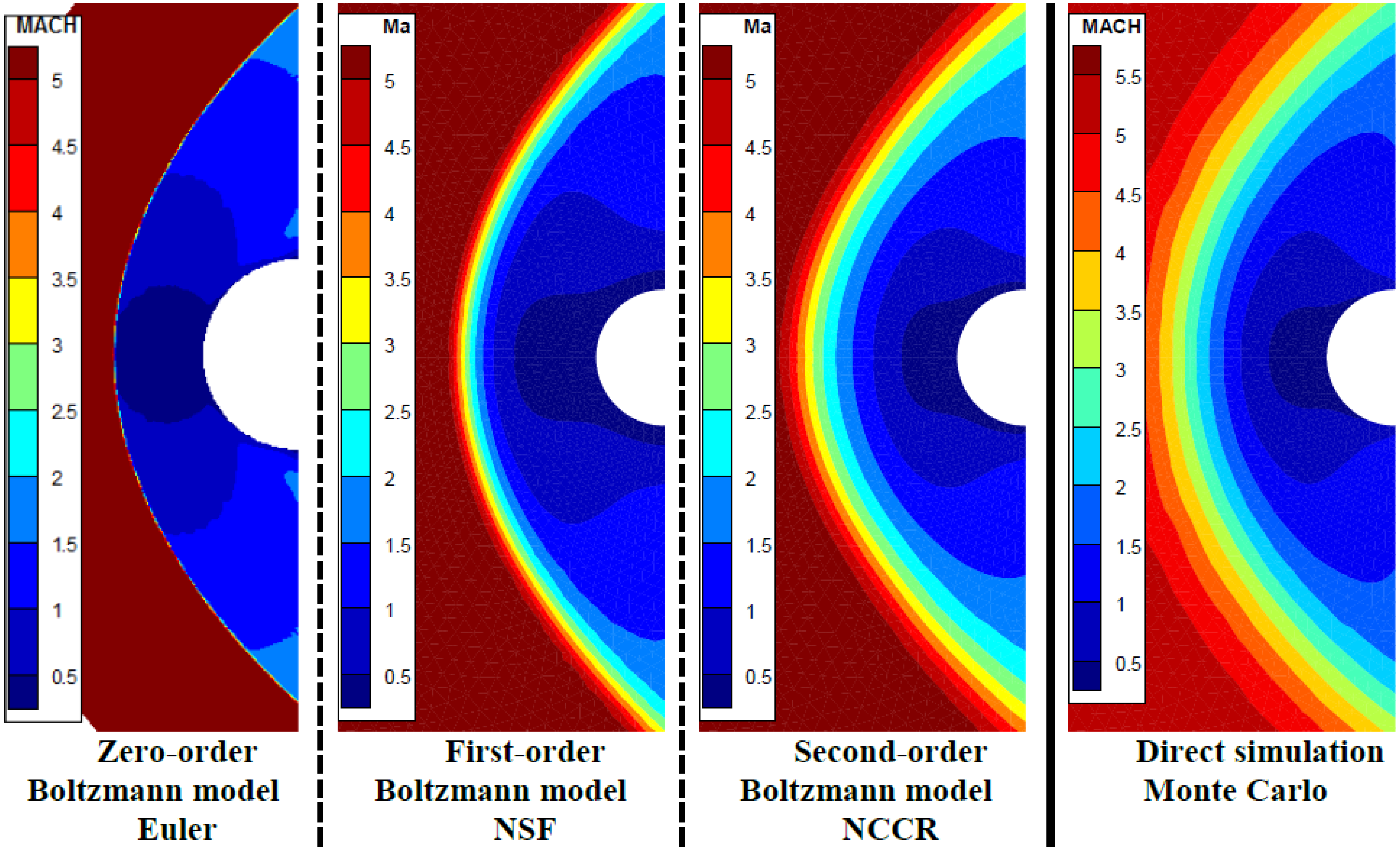
Convergence property

NCCR: Nonlinear Coupled Constitutive Relation



Super-parallel performance (via the rate of cost reduction) (C&F, 2017)

Two-dimensional shock structure around cylinder



Computed Mach contours over cylinder (Mach 5.48, Kn=0.2)

NCCR: Nonlinear Coupled Constitutive Relation

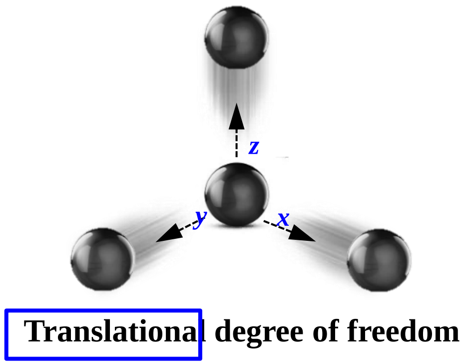
(Karchani, *PhD Thesis*, GNU, 2017)

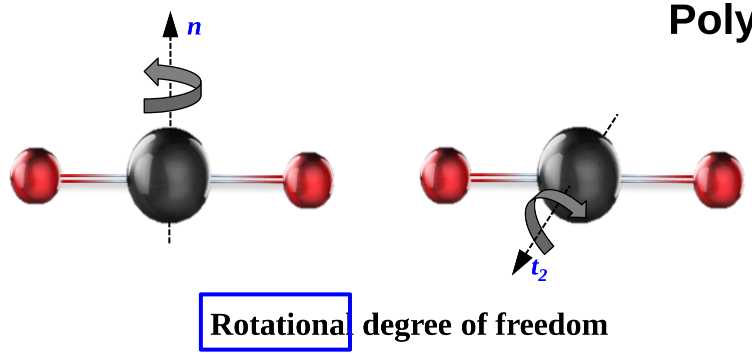
Monatomic, diatomic, and (linear) polyatomic gases

Monatomic

Diatomic

Polyatomic





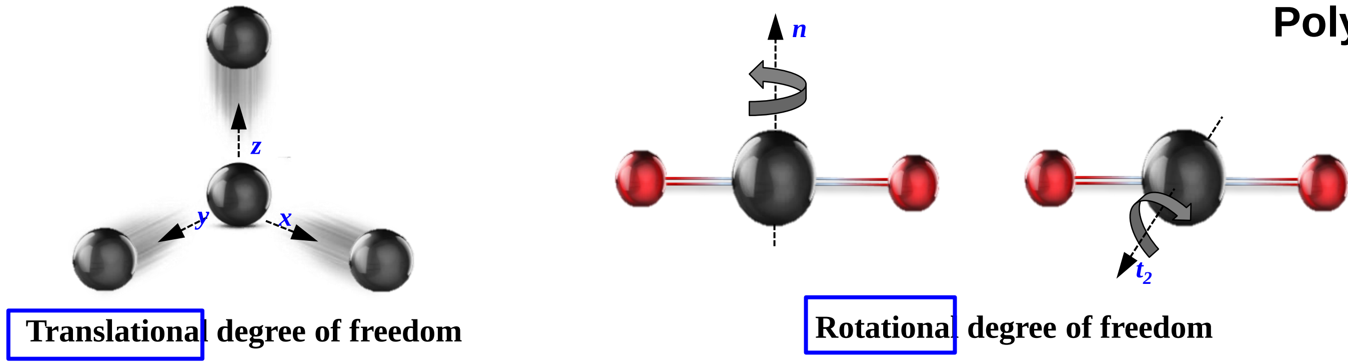
- Once the Stokes assumption is abandoned, an additional constitutive law of the excess normal stress Δ related to the bulk viscosity will appear

$$\Delta = \mu_b(\Delta \cdot \boldsymbol{u}).$$

- The inner structure of strong shock waves (Emanuel and Argrow 1994)
- Dilatational waves in transition in hypersonic boundary layers (Zhu et al. 2016)

Monatomic, diatomic, and (linear) polyatomic gases

- Monatomic
- Diatomic
- Polyatomic



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Boltzmann-Curtiss equation for polyatomic gases

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla+\boxed{\frac{j}{I}\frac{\partial}{\partial\psi}}\right)f\left(\mathbf{v},\mathbf{r},\boxed{\mathbf{j},\psi},t\right)=C\left[f,f_2\right]$$

$$\rho\mathbf{u}\equiv\left\langle m\mathbf{v}f\left(\mathbf{v},\mathbf{r},\boxed{\mathbf{j},\psi},t\right)\right\rangle$$

I = moment of inertia
j = angular momentum
ψ= azimuthal angle

$$\rho\equiv\left\langle m\,f\left(\mathbf{v},\mathbf{r},\mathbf{j},\psi,t\right)\right\rangle$$

Rotational energy

$$\rho E\equiv\left\langle\left(\frac{1}{2}mc^2+\boxed{H_{rot}}\right)f\left(\mathbf{v},\mathbf{r},\boxed{\mathbf{j},\psi},t\right)\right\rangle$$

$$\langle\ldots\rangle\equiv\int\!\!\int\!\!\int\ldots dv_x dv_y dv_z\,\boxed{jdjd\psi}\,d\Omega$$

$$\rho\frac{d}{dt}\begin{bmatrix}1/\rho\\ \mathbf{u}\\ E_t\end{bmatrix}+\nabla\cdot\begin{bmatrix}-\mathbf{u}\\ p\mathbf{I}\\ p\mathbf{u}\end{bmatrix}+\nabla\cdot\begin{bmatrix}0\\ \mathbf{\Pi}+\boxed{\Delta\mathbf{I}}\\ (\mathbf{\Pi}+\boxed{\Delta\mathbf{I}})\cdot\mathbf{u}+\mathbf{Q}\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}.$$

$\mathbf{\Pi}$: normal stress tensor $=-2\mu_{NS}\left[\nabla\mathbf{u}\right]^{(2)}$

Δ : excess normal stress $=-\mu_b\nabla\cdot\mathbf{u}$

$\mathbf{Q}$: heat flux vector $=-k\nabla T$

$$\Delta=\left\langle m\mathrm{Tr}(\mathbf{cc})f/3-m\mathrm{Tr}(\mathbf{cc})f^{(0)}/3\right\rangle,\ p=\left\langle m\mathrm{Tr}(\mathbf{cc})f^{(0)}/3\right\rangle$$

2nd-order constitutive relations for polyatomic gases

$$\boldsymbol{\Pi} = \left\langle h^{(1)} f \right\rangle \qquad h^{(1)} = m\left[\mathbf{cc}\right]^{(2)}$$

$$\Delta = \left\langle h^{(2)} f \right\rangle \qquad h^{(2)} = mc^2/3 - p/n$$

$$\mathbf{Q} = \left\langle h^{(3)} f \right\rangle \qquad h^{(3)} = \left[mc^2/3 + H_{rot} - m\hat{h} \right]\mathbf{c}$$

2nd-order

$$\boxed{\begin{aligned} &[\hat{\boldsymbol{\Pi}}\cdot\nabla\hat{\mathbf{u}}]^{(2)} + \left(f_b\hat{\Delta}+1\right)\hat{\boldsymbol{\Pi}}_{NF} = \hat{\boldsymbol{\Pi}}\,q_{2nd}(c\hat{R}) \\ &\frac{3}{2}f_b\left(\hat{\boldsymbol{\Pi}}+f_b\hat{\Delta}\mathbf{I}\right):\nabla\hat{\mathbf{u}} + \hat{\Delta}_{NF} = \hat{\Delta}\,q_{2nd}(c\hat{R}) \\ &\hat{\boldsymbol{\Pi}}\cdot\hat{\mathbf{Q}}_{NS} + \left(f_b\hat{\Delta}+1\right)\hat{\mathbf{Q}}_{NF} = \hat{\mathbf{Q}}\,q_{2nd}(c\hat{R}) \end{aligned}}$$

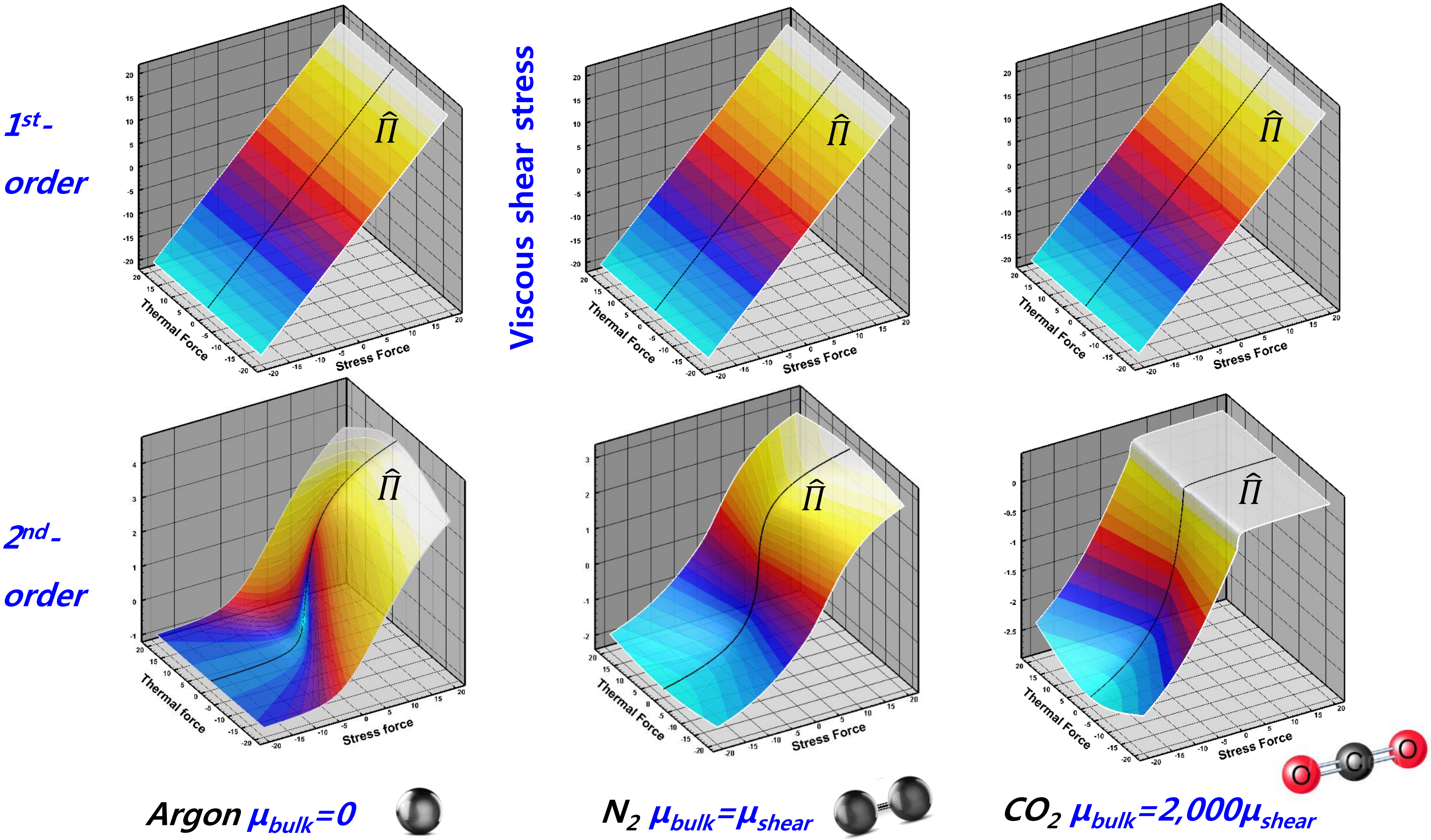
$\Longrightarrow$ *Linearization*

1st-order

$$\boxed{\begin{aligned} &\hat{\boldsymbol{\Pi}}_{NS} = -2\mu\left[\nabla\mathbf{u}\right]^{(2)} \\ &\hat{\Delta}_{NS} = -\mu_b\nabla\cdot\mathbf{u} \\ &\hat{\mathbf{Q}}_{NS} = -k\nabla T \end{aligned}}$$

Topological representation of constitutive relations is possible!

Topology of 2nd-order CR (compression/expansion)



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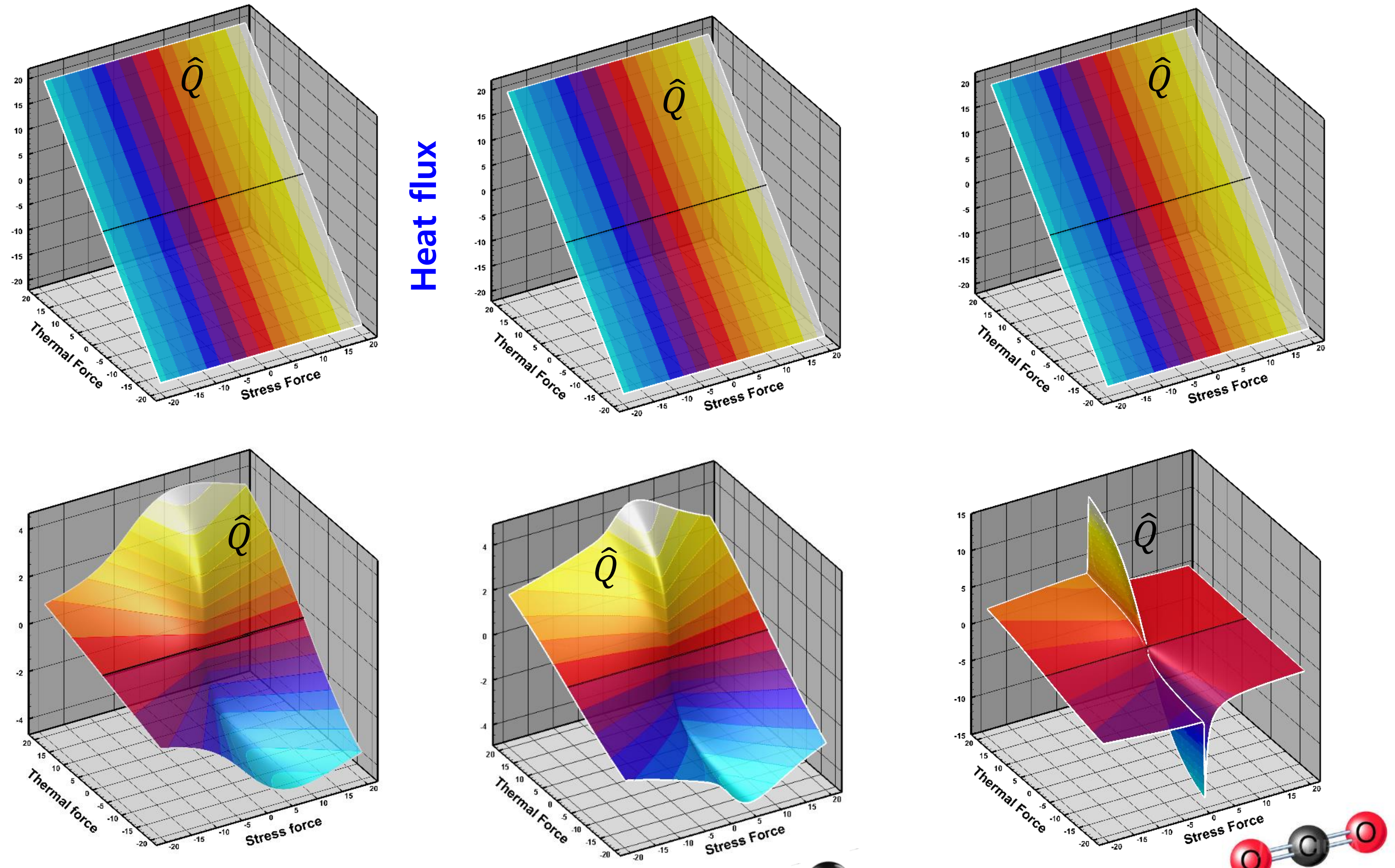
Topology of 2nd-order CR (compression/expansion)

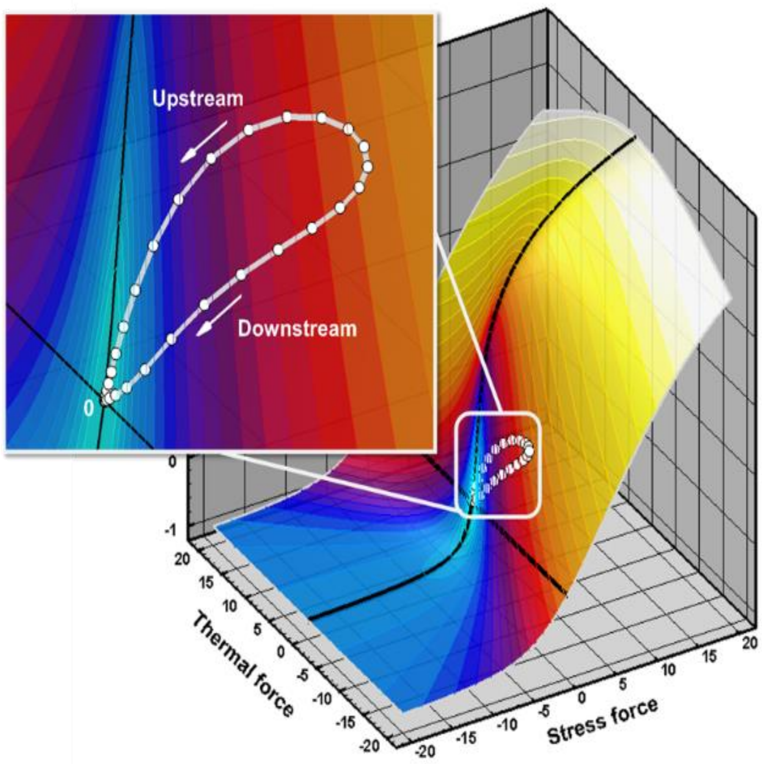
1st-order

2nd-order

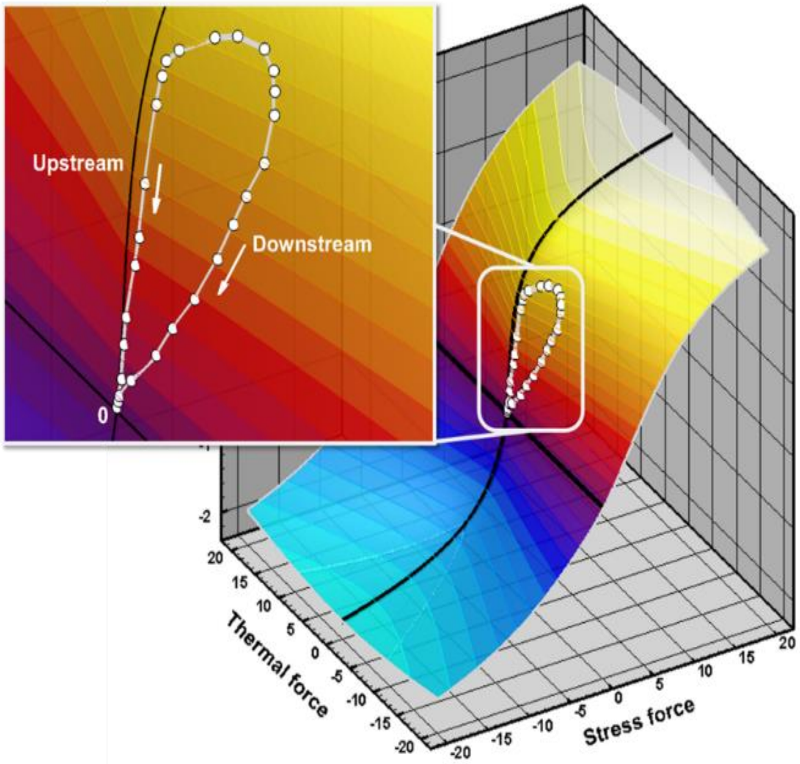


Argon $\mu_{bulk}=0$ ***N_2*** $\mu_{bulk}=\mu_{shear}$ ***CO_2*** $\mu_{bulk}=2{,}000\mu_{shear}$

Trajectory of shock structure in topology of CR



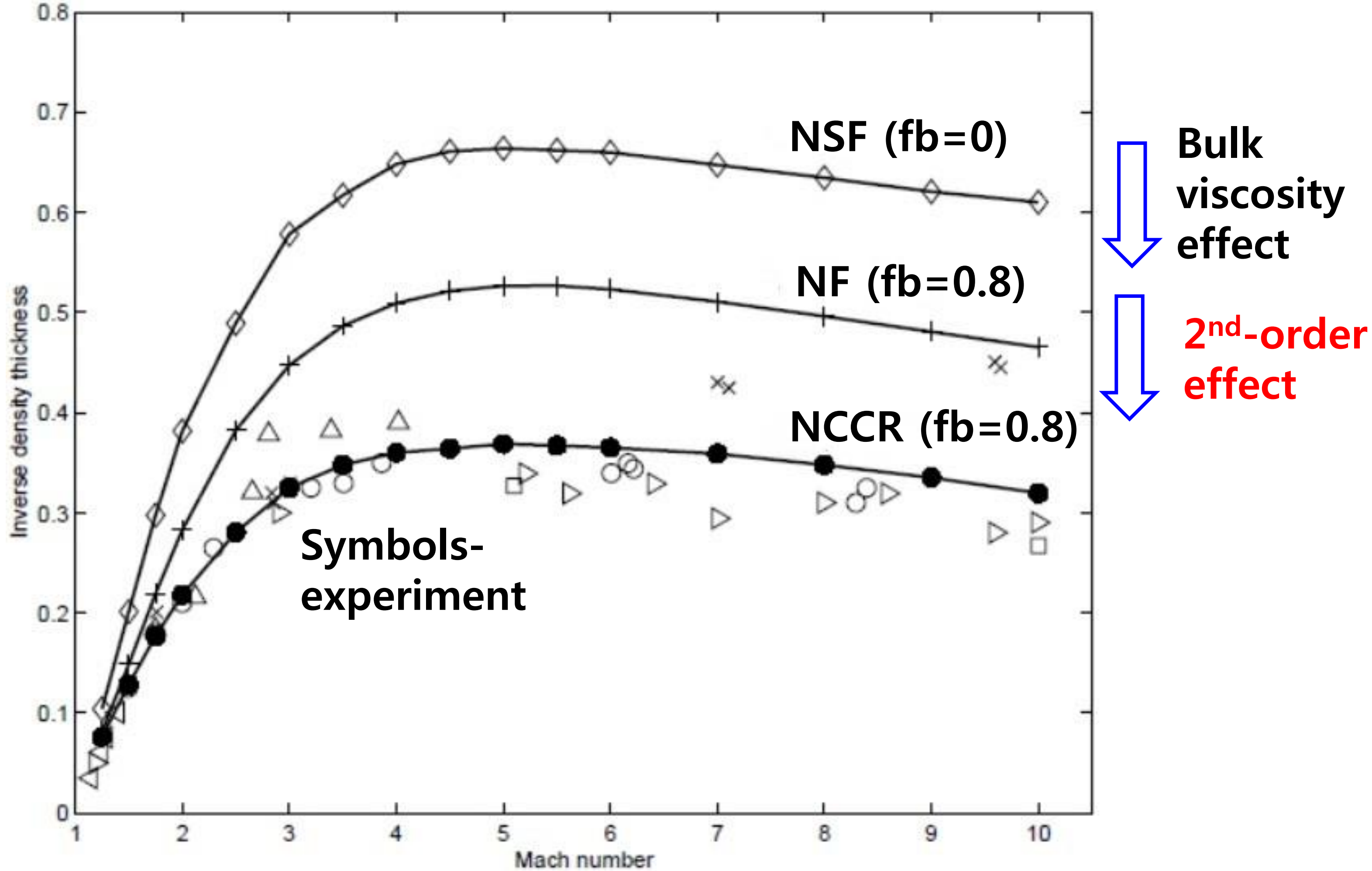
f_b=0.0



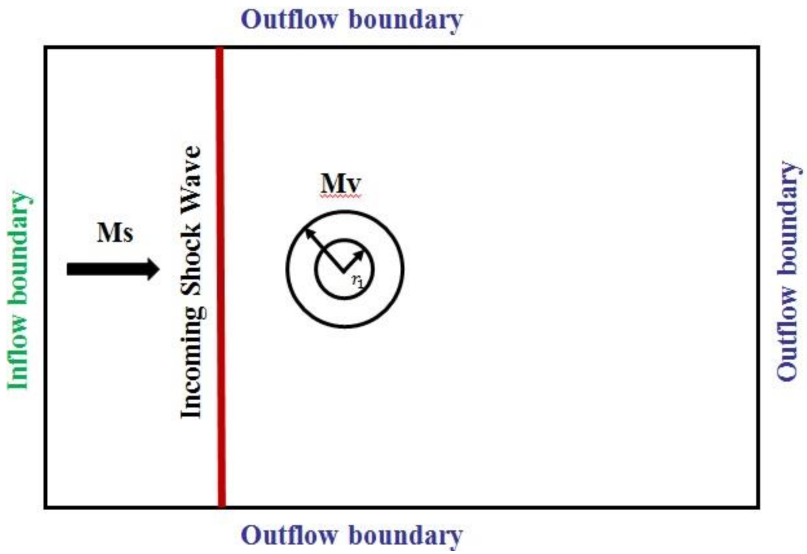
f_b=0.8

Mach number = 5.0

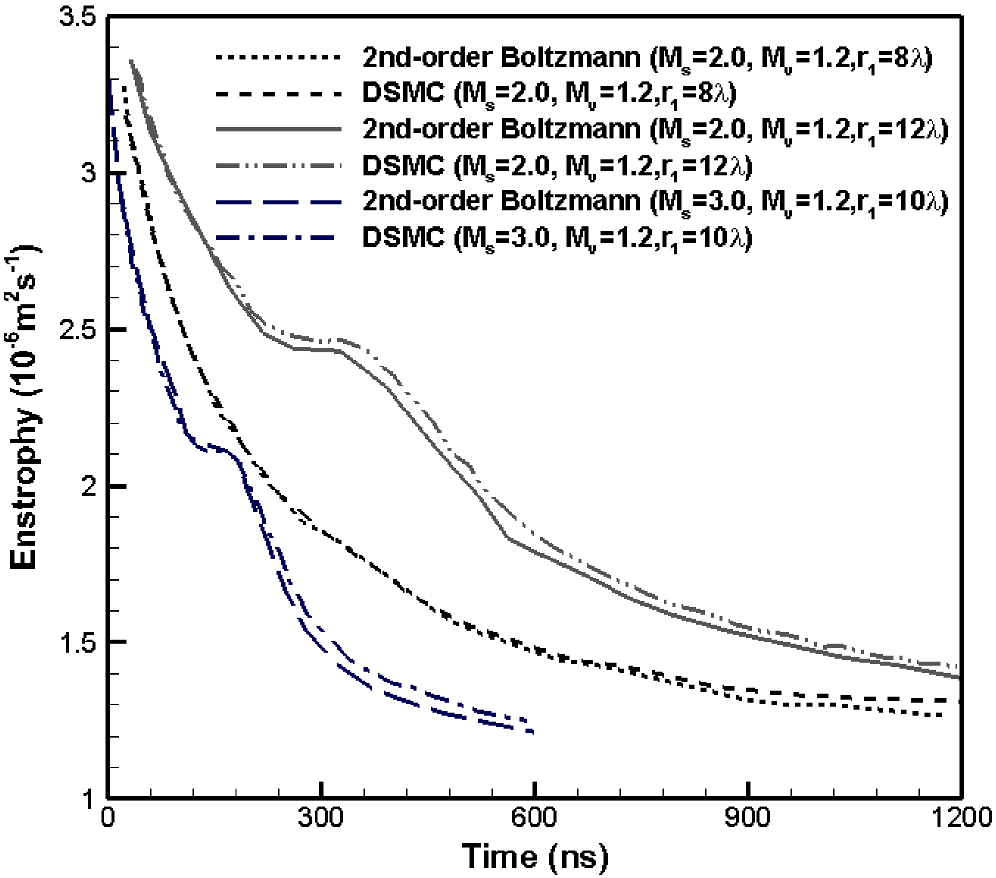
Inverse density thickness of shock wave (nitrogen)



Multi-dimensional case: shock-vortex interaction



PoF 2018



$$\text{Enstrophy}(t) = \int_{\partial A} \Omega_z^2(x,y,t)\,dxdy,$$

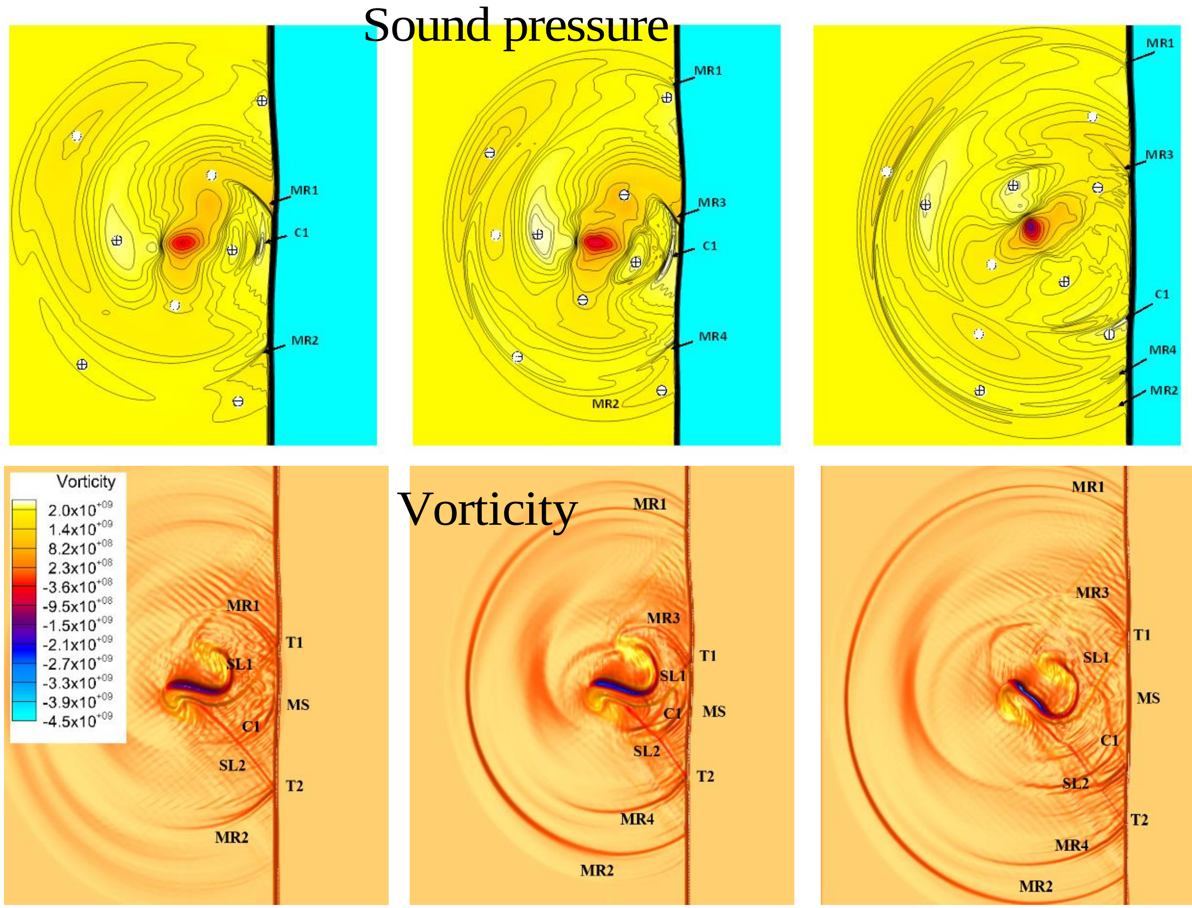
$$\text{Vorticity } \Omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

Multi-dimensional case: shock-vortex interaction

Shock Mach = 2.0

Vortex Mach = 0.8

Knudsen =0.001

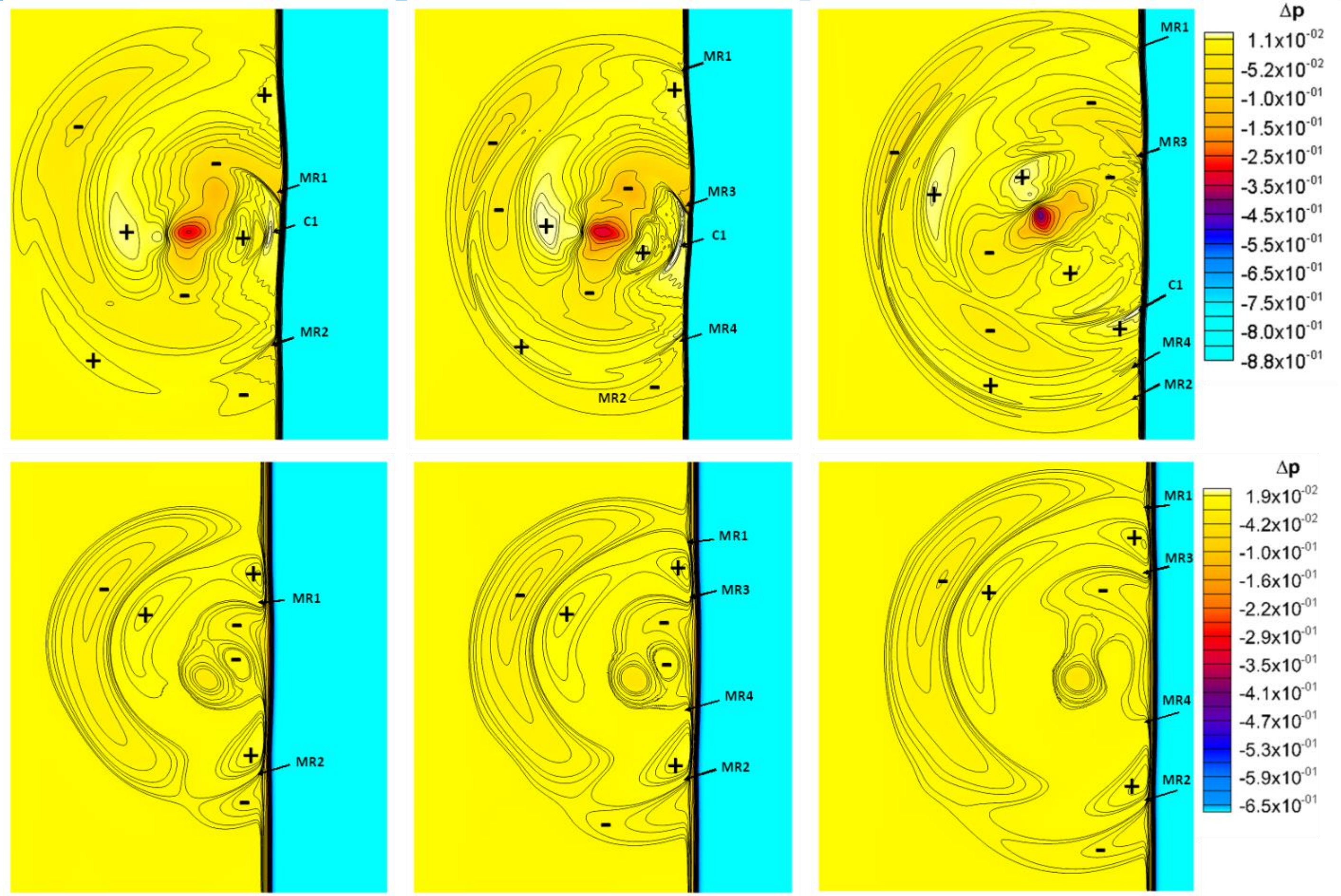


Argon	Nitrogen	Methane
$f_b = 0$	$f_b = 0.75$	$f_b = 1.33$
$\gamma = 1.667$	$\gamma = 1.4$	$\gamma = 1.289$

Multi-dimensional case: shock-vortex interaction

Macro SVI

Micro SVI



(a) Argon gas, $f_b = 0.0$ (b) Nitrogen gas, $f_b = 0.8$ (c) Methane gas, $f_b = 1.33$

Effects of diatomic and polyatomic gases on macro SVI with $M_s = 2.0, M_v = 0.8,\ r_1 = 1000\lambda$ (top) and micro SVI with $M_s = 2.0, M_v = 0.8,\ r_1 = 10\lambda$ (bottom): sound pressure at $t = 1000\,\text{ns}$.

Dusty gas flows in Lunar landing