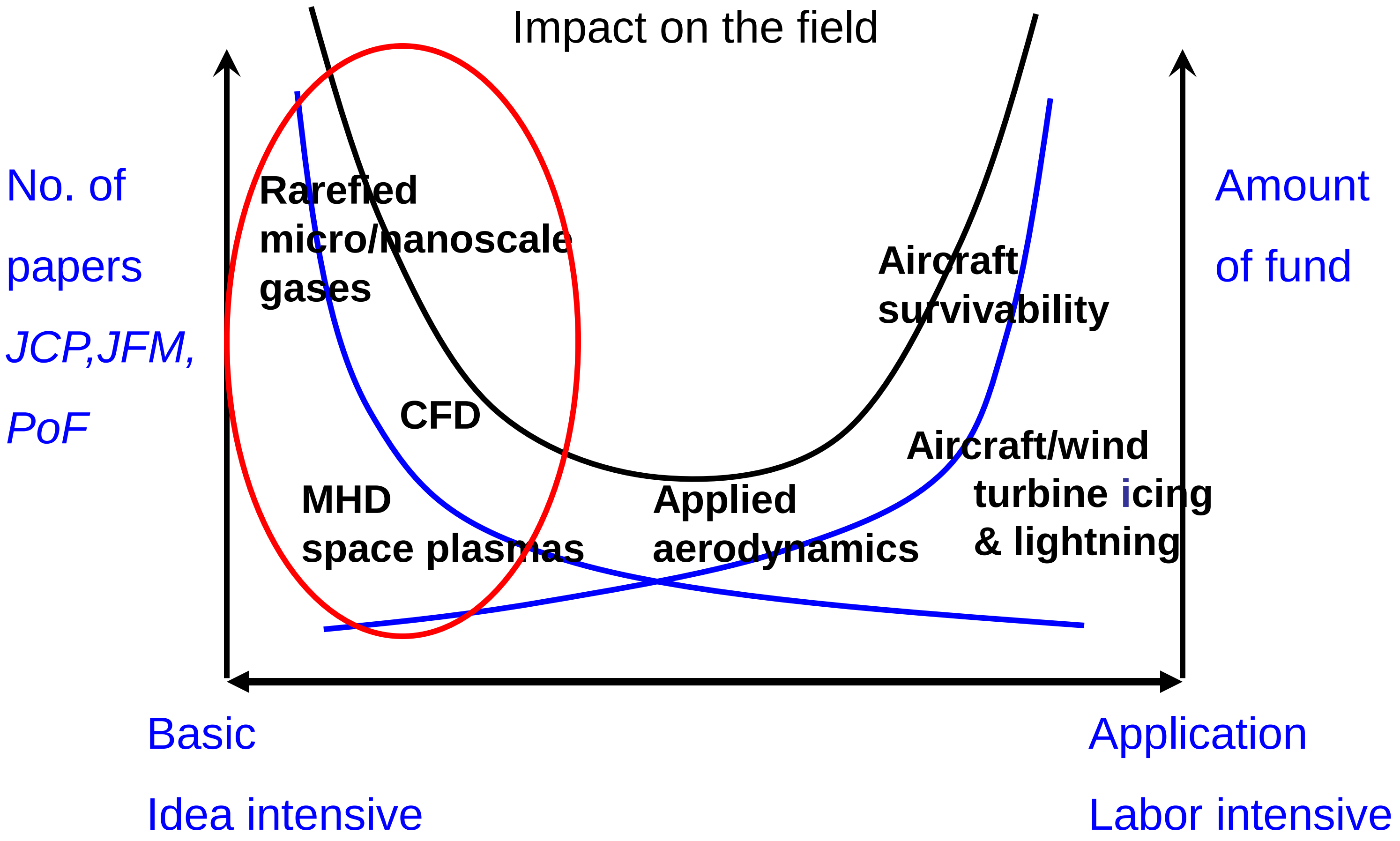
Contributions to Fundamentals of MHD and Gases in Thermal Nonequilibrium

via

Mathematical Equations
and
Figures

December 20, 2013

Basic Research

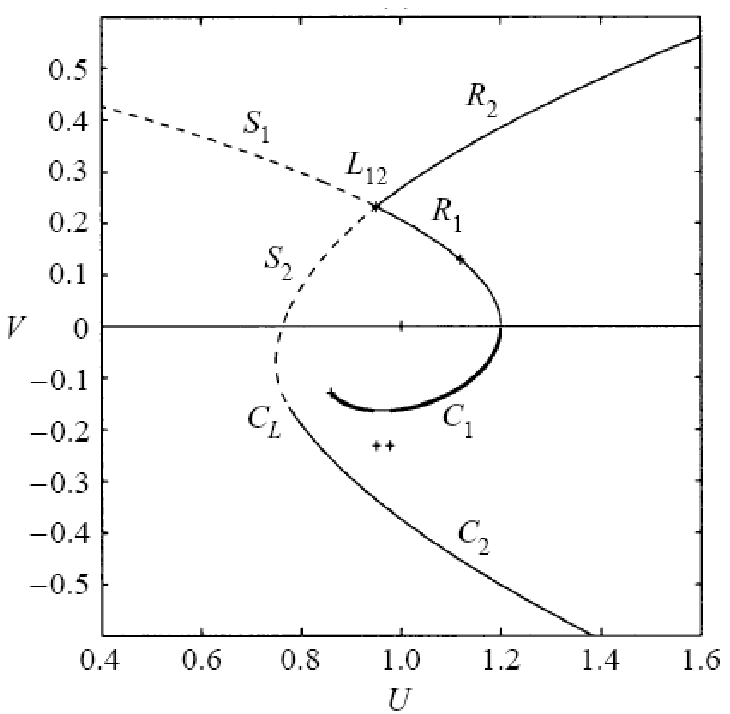


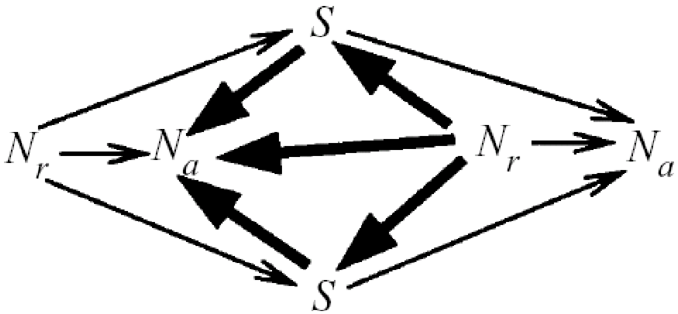
MHD: Shocks and Riemann Problem I

Planar Riemann problem is well-posed when intermediate shocks are included.

$$\left([U]+\gamma\overline{V}[V]\right)\left\{\gamma(\gamma-1)[V]^3+4\gamma\overline{U}[V]-4\overline{V}[U]\right\}-4\gamma[U][V]=0$$

$$U\equiv\gamma p/B_x^2,\ V\equiv B_y/B_x:\quad \text{MHD Rankine-Hugoniot}$$



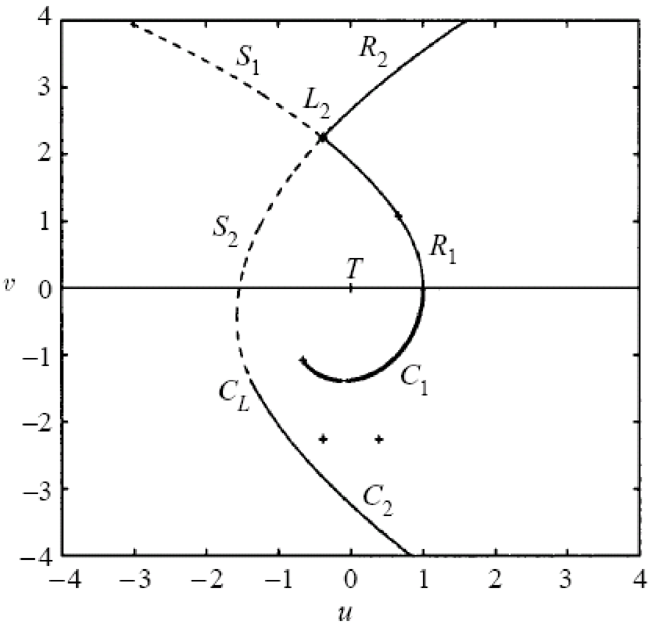


MHD: Shocks and Riemann Problem II

A 3*3 model problem exactly preserving the MHD hyperbolic singularities

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix}_t + \begin{bmatrix} (\gamma+1)u^2 + v^2 + w^2 \\ 2uv \\ 2uw \end{bmatrix}_x = \begin{bmatrix} \mu & 0 & 0 \\ 0 & \eta & -\chi \\ 0 & \chi & \eta \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}_{xx}$$

$$u \equiv \gamma p / B_x^2 - 1,\ v \equiv B_y / B_x,\ w \equiv B_z / B_x$$



An overcompressive shock structure

$$u(x) = -u_0 \tanh(2u_0 x / \mu),$$

$$v(x) = -v_0 \tanh(2u_0 x / \mu) + C w_0 \operatorname{sech}(2u_0 x / \mu),$$

$$w(x) = -w_0 \tanh(2u_0 x / \mu) - C v_0 \operatorname{sech}(2u_0 x / \mu),$$

$$C^2 \equiv 1 + (\gamma - 1) u_0^2 / (v_0^2 + w_0^2)$$

Thermodynamically-Consistent Algebraic Constitutive Relation for Rarefied and Microscale Gases beyond Navier-Stokes-Fourier

$$0 = -\begin{bmatrix} 2\left[\mathbf{\Pi}\cdot\nabla\mathbf{u}\right]^{(2)} \\ C_p\mathbf{\Pi}\cdot\nabla T + \mathbf{Q}\cdot\nabla\mathbf{u} + \dfrac{D\mathbf{u}}{Dt}\cdot\mathbf{\Pi} \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{a}\cdot\mathbf{\Pi} \end{bmatrix} - \begin{bmatrix} 2p\left[\nabla\mathbf{u}\right]^{(2)} \\ C_p p\nabla T \end{bmatrix} + \begin{bmatrix} \mathbf{\Lambda}^{(\Pi)} \\ \mathbf{\Lambda}^{(Q)} \end{bmatrix}$$

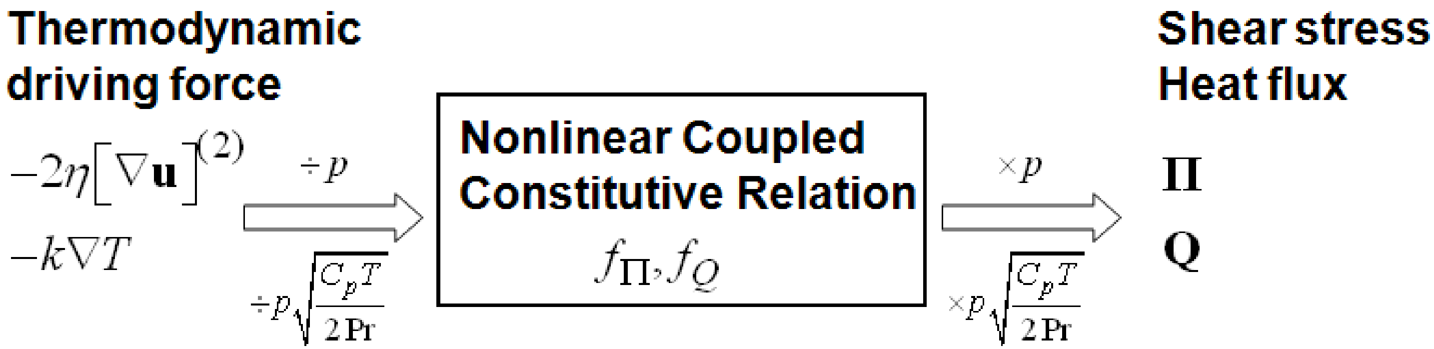
Kinematic **Force** **Thermo. driving** **Dissipation**

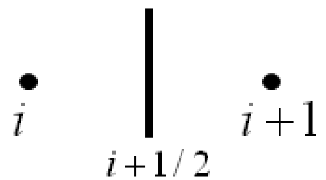
$$\mathbf{\Lambda}^{(\Pi)} = -\frac{\mathbf{\Pi}}{\eta / p} q(\kappa),\ \ \mathbf{\Lambda}^{(Q)} = -\frac{\mathbf{Q}}{k / pC_p} q(\kappa),$$

$$q(\kappa) \equiv \frac{\mathbf{sinh}\kappa}{\kappa} \ \text{ where } \ \kappa = \frac{\left(mk_BT\right)^{1/4}}{\sqrt{2}pd}\left(\frac{\mathbf{\Pi}:\mathbf{\Pi}}{2\eta} + \frac{\mathbf{Q}\cdot\mathbf{Q}}{k/T}\right)^{1/2},$$

$$\sigma_{\text{entropyproduction}} = -k_B < \ln FC[F, F_2] >= k_B\,\frac{\left(m/2k_BT\right)^{1/2}}{n^2d^2}\,\kappa\mathbf{sinh}\kappa \geq 0$$

A Computational Framework Based on NCCR





$$\mathbf{\Pi}_{i+1/2_{NSF}} = -2\eta_{i+1/2}\left[\nabla\mathbf{u}\right]^{(2)}_{i+1/2}$$

$$\mathbf{Q}_{i+1/2_{NSF}} = -k_{i+1/2}(\nabla T)_{i+1/2}$$

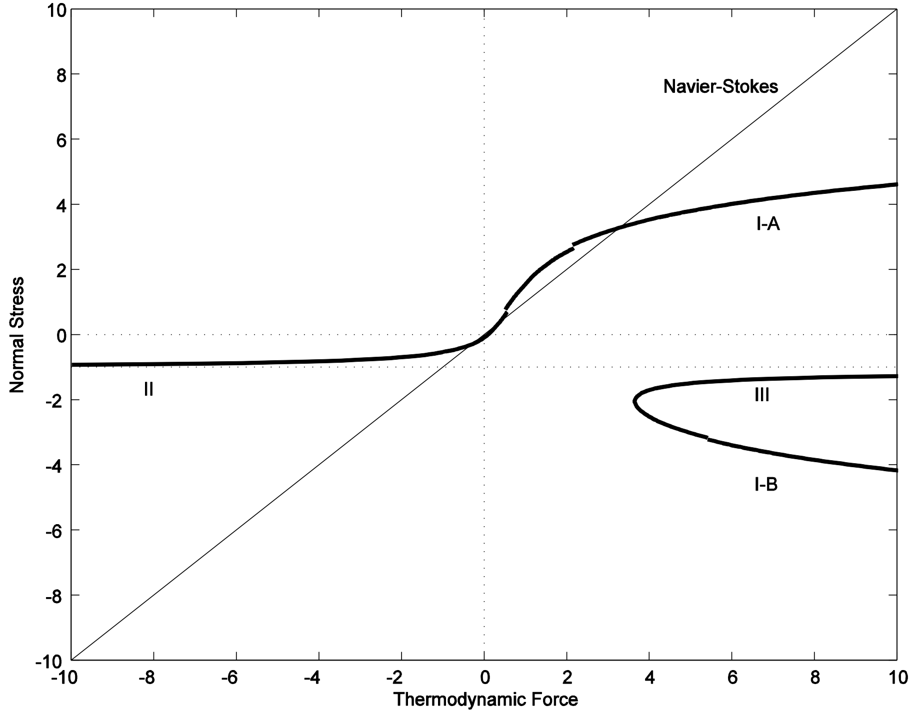
$$\mathbf{\Pi}_{i+1/2_{NCCR}} = f_\Pi\left(\mathbf{\Pi}_{i+1/2_{NSF}}, \mathbf{Q}_{i+1/2_{NSF}}, p_{i+1/2}, T_{i+1/2}\right)$$

$$\mathbf{Q}_{i+1/2_{NCCR}} = f_Q\left(\mathbf{\Pi}_{i+1/2_{NSF}}, \mathbf{Q}_{i+1/2_{NSF}}, p_{i+1/2}, T_{i+1/2}\right)$$

A Model Exactly Preserving NCCR Nonlinearity: NCCR-Burgers

$$\frac{\partial u}{\partial t}+\frac{\partial (u^2/2)}{\partial x}=-\frac{\partial \hat{\Pi}}{\partial x}\ , \quad a\le x\le b, \quad t\ge 0,$$

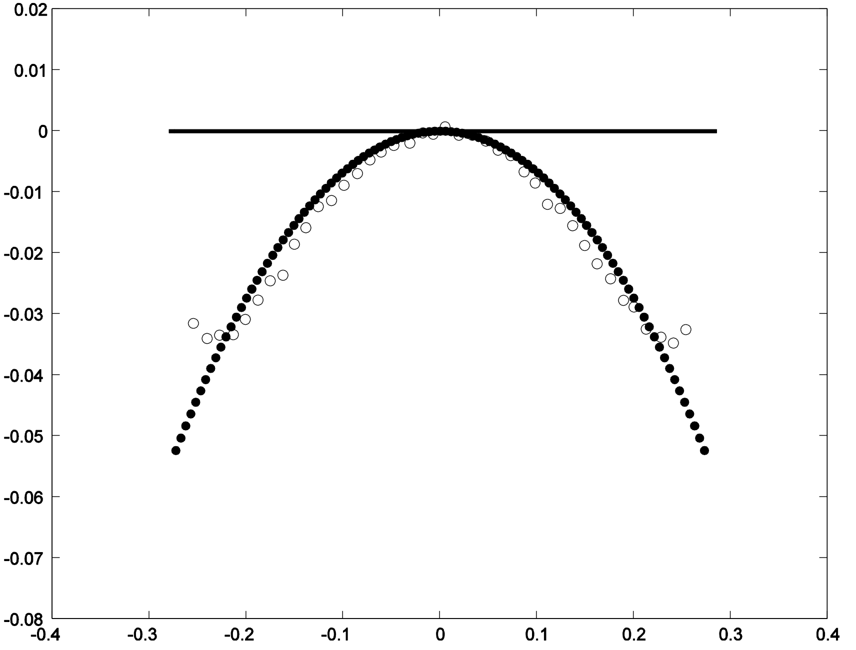
$$\hat{\Pi}q(\hat{\Pi})=(1+\hat{\Pi})\hat{\Pi}_{NS}\,,\ \text{where}\ \ q(\hat{\Pi})\equiv\frac{\sinh\hat{\Pi}}{\hat{\Pi}},\ \ \hat{\Pi}_{NS}\equiv-\mu\frac{\partial u}{\partial x}$$



Kinematic Stress Constraint and Cross-Stream Pressure Variation in Nonequilibrium Gases

$$\left(\frac{\Pi_{xy}}{p}\right)^2 = -\frac{3}{2}\left(1+\frac{\Pi_{yy}}{p}\right)\frac{\Pi_{yy}}{p}$$

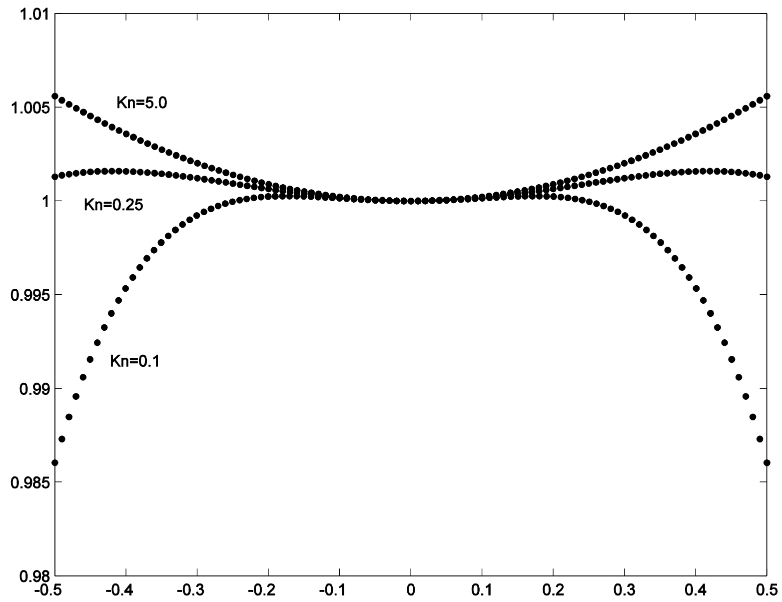
$$\frac{p(S^*)}{p(0)} = 1 + \tan^2 S^*$$



Central Temperature Minimum in Force-Driven Poiseuille Gas Flow

$$T^*(S^*) = \sec^e S^* \left\{ T^*(0) - \left[T^*(0) - \sec^{-e} S^*_{1/2} \left(\alpha_T T^*_w + \left(1-\alpha_T\right) T^*(0) \right) \right] \frac{F(S^*)}{F(S^*_{1/2})} \right\},$$

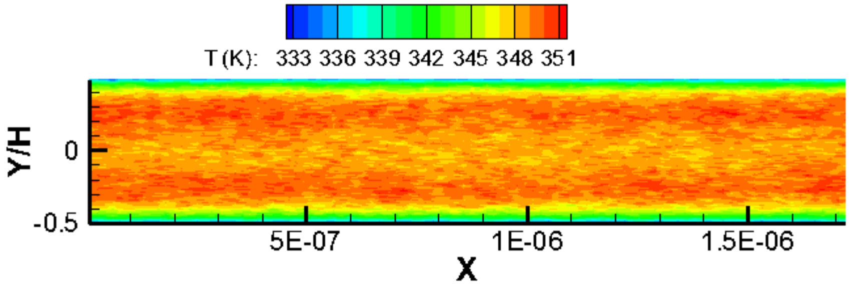
$$F\left(S^*\right) \equiv (4-e)\left[\frac{1}{(4-e)\cos^{4-e} S^*} - \frac{1}{(2-e)\cos^{2-e} S^*} - \left(\frac{1}{4-e} - \frac{1}{2-e} \right) \right], \quad e = \frac{3\left(\gamma - 1\right)}{2\gamma}$$



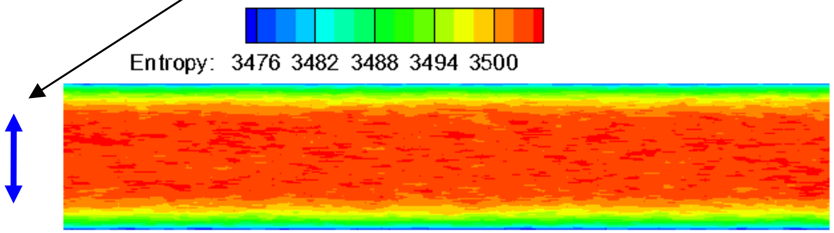
An Exotic Prediction: Heat Conduction from Cold to Hot

Non-Fourier behavior in NCCR

$$\hat{Q}_y = \frac{3}{\left(3+2\hat{\Pi}^2_{xy_{NSF}}\right)}\left(\hat{Q}_{y_{NSF}} + \hat{a}\hat{\Pi}_{xy_{NSF}}\right);\ a \text{ is force}$$



Temperature contours



Heat transfer from

cold (center) to hot (wall)

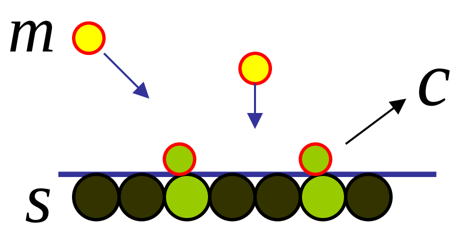
H function contours

Boundary Condition Based on Langmuir Adsorption Isotherm

Fraction of sites in thermal equilibrium α

$$K = \frac{C_c}{C_m C_s} = \frac{N\alpha}{[p / k_B T_w] N(1-\alpha)},$$

$$\alpha = \frac{\beta p}{1+\beta p} \quad \text{where} \quad \beta = \frac{K}{k_B T_w}.$$



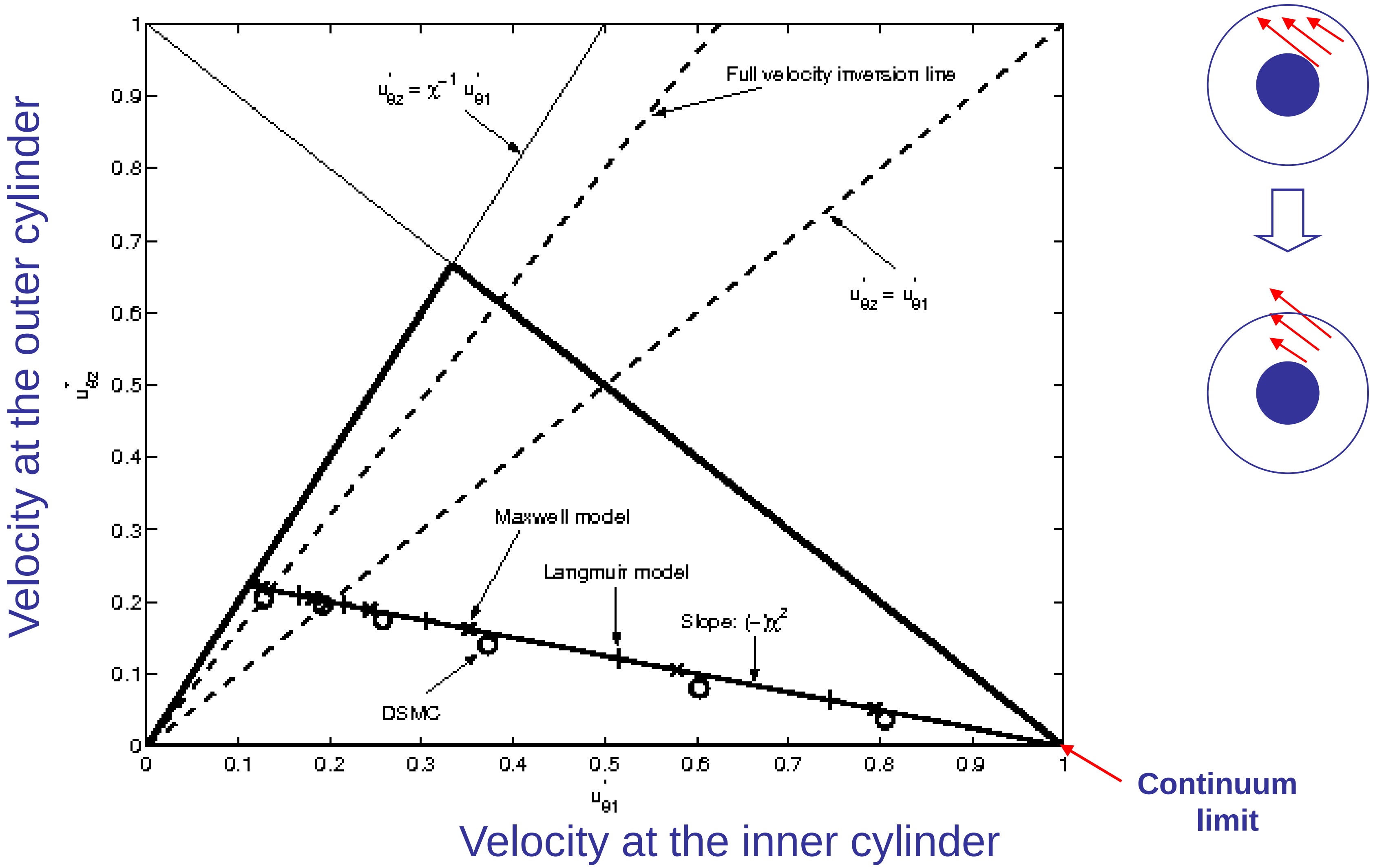
$m + s \Rightarrow c$

Maxwell accommodation coefficients $\sigma_{v,T}$

$$\sigma_{v,T} \sim \omega_0(v) \left(\frac{T_w}{T_r} \right)^{1+2/(v-1)} \exp\left(-\frac{D_e}{k_B T_w} \right)$$

D_e : Heat of adsorption $[O(10^{-1} \sim 10)$ kcal/mol$]$

Microscale Cylindrical Couette Gas Flow



Microscale Cylindrical Couette Gas Flow

If the radius ratio is equal to the golden ratio $(3-\sqrt{5})/2$, Maxwell and Langmuir models coincide.

Gaseous Slip on Microscale Heat Transfer (An Extended Graetz Problem)

$$(1-r^2)\frac{\partial\theta}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) + \frac{\alpha(2-\alpha)^2}{Pe^2}\frac{\partial^2\theta}{\partial z^2}$$

(Confluent hypergeometric function of the first kind and finite integral transform technique)

The Reynolds analogy between heat transfer and momentum transfer is preserved in slip-flow regimes with low Mach numbers.

Analytical Solutions of Shock Structure Thickness and Asymmetry in Navier-Stokes-Fourier Framework (Pr=3/4)

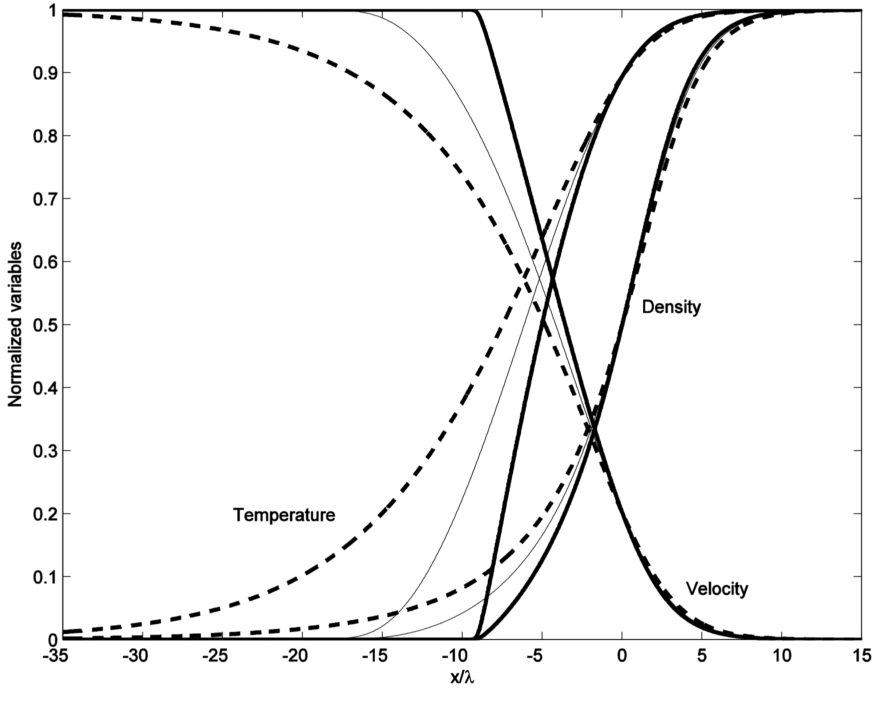
Asymmetry of Maxwellian molecule

$$Q_s = \frac{\int_{-\infty}^{0}\left(r(x)-r_1\right)dx}{\int_{0}^{\infty}\left(r_2-r(x)\right)dx} = \frac{\int_{r_a}^{r_1} x(r)dr}{\int_{r_a}^{r_2} x(r)dr} =$$

$$\frac{-\frac{\mu}{4}\left[\frac{5}{4}\ln\frac{r_1}{r_a}-\frac{1}{2}\left(\frac{1}{r_1}-\frac{1}{r_a}\right)-\frac{1}{r_a}\left(\frac{5}{4}+\frac{1}{2r_a}\right)(r_1-r_a)\right]-\frac{5}{4}\left(\frac{1}{r_2}-\frac{1}{r_1}\right)\ln\frac{r_1}{r_a}-\left(2\alpha-\frac{1}{r_2^2}\right)\left(1-\frac{r_1}{r_2}\right)\ln\left(\frac{r_a^{-1}-r_2^{-1}}{r_1^{-1}-r_2^{-1}}\right)}{-\frac{\mu}{4}\left[\frac{5}{4}\ln\frac{r_2}{r_a}-\frac{1}{2}\left(\frac{1}{r_2}-\frac{1}{r_a}\right)-\frac{1}{r_a}\left(\frac{5}{4}+\frac{1}{2r_a}\right)(r_2-r_a)\right]-\frac{5}{4}\left(\frac{1}{r_2}-\frac{1}{r_1}\right)\ln\frac{r_2}{r_a}-\left(2\alpha-\frac{1}{r_1^2}\right)\left(\frac{r_2}{r_1}-1\right)\ln\left(\frac{r_1^{-1}-r_a^{-1}}{r_1^{-1}-r_2^{-1}}\right)}$$

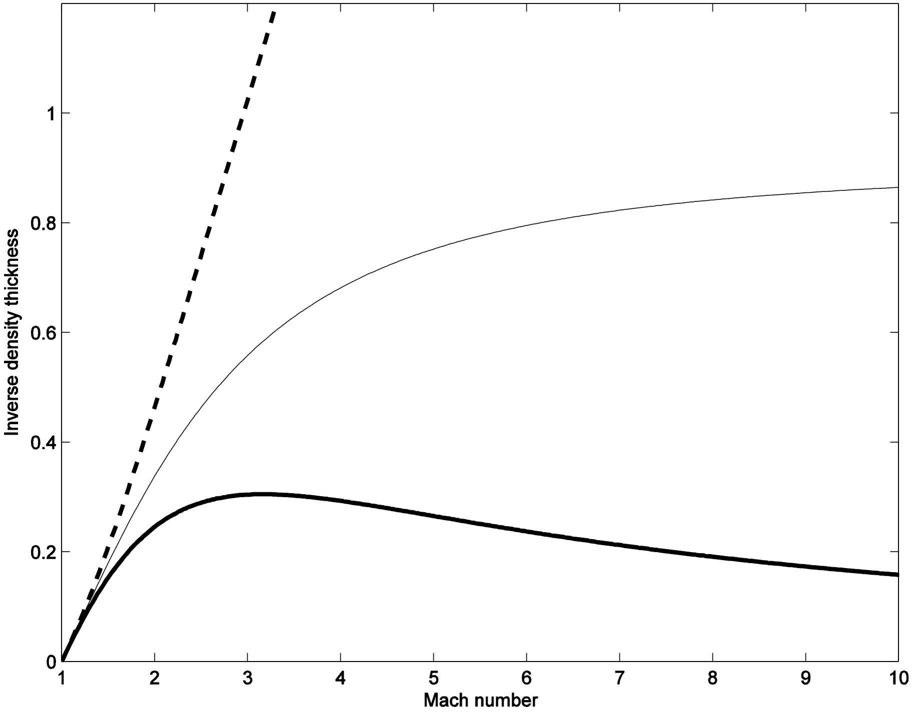
$$Q_{s_{M\to 1}} = 1, \quad Q_{s_{M\to\infty}} = \frac{125}{106}\ln 5 - \frac{38}{53} \doteq 1.181.$$

Analytical Solutions of Shock Structure Thickness and Asymmetry in Navier-Stokes-Fourier Framework (Pr=3/4)



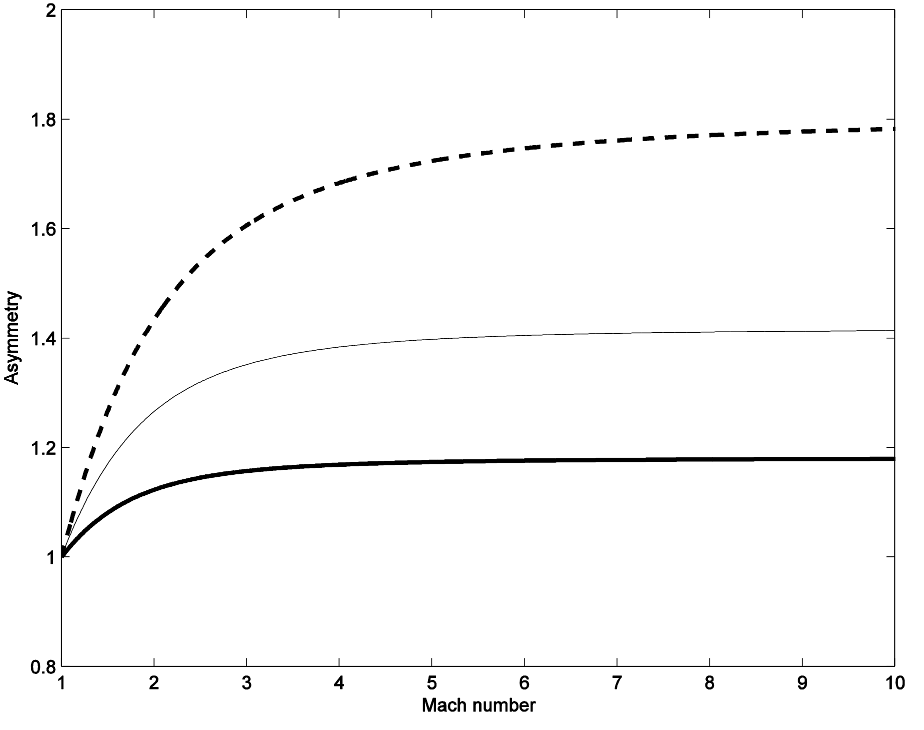
Shock structure profiles (*M*=15). Maxwellian molecule (*s*=1) for the thick solid curve; hard sphere (*s*=1/2) for the thin solid curve; constant case (*s*=0) for the broken curve

Analytical Solutions of Shock Structure Thickness and Asymmetry in Navier-Stokes-Fourier Framework (Pr=3/4)



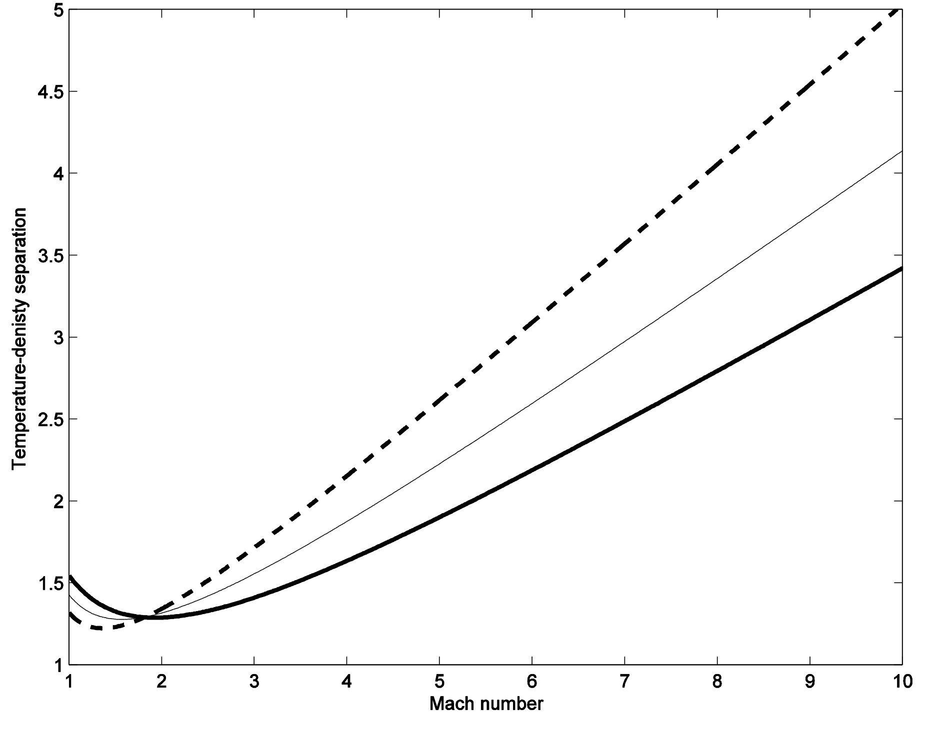
Inverse shock density thickness

Analytical Solutions of Shock Structure Thickness and Asymmetry in Navier-Stokes-Fourier Framework (Pr=3/4)



Shock asymmetry

Analytical Solutions of Shock Structure Thickness and Asymmetry in Navier-Stokes-Fourier Framework (Pr=3/4)



Shock temperature–density separation

$$\Delta_{M\to 1} = \frac{4}{5}\sqrt{\frac{10}{\pi}} \doteq 1.427, \quad \Delta_{M\to\infty} \to \infty \text{ for hard sphere}$$